Judicial syllogism - integrating non-monotonic logic in a deductive logical form

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Abstract

The judicial syllogism represents one of the most significant and widely accepted applications of logic in the field of law. Alongside the legislative syllogism, it is a part of the broader conceptual framework commonly referred to as the legal syllogism. This logical structure is classified as a type of mediate deductive inference, which proceeds from general to particular statements – a reasoning process traditionally associated with the dictum de omni principle. Although intuitive and traditionally accepted for offering the proper structure for the application of law, the judicial syllogism is inherently static due to its foundation in classical mediate deductive reasoning. It does not accommodate the dynamic nature of judicial processes, where addressing quaestio juris and quaestio facti may modify the premises, thereby altering the conclusions. Therefore, this article intends to analyse the classical types of mediate deductive inferences, the static nature of the judicial syllogism and the shortcomings of this monotonic type of logic where the conclusion does not change once it is derived. After analysing the points in the construction of the judicial syllogism, the article also proposes a way of integrating non-monotonic logic in the elaboration of the judicial syllogism in order to capture the actual dynamic of judiciary processes of applying law to particular cases, without altering the overall structure of the judicial syllogism. The operationalization of this theoretical framework could be of practical relevance in developing computational tools, especially in AI applications.

Keywords: legal syllogism, judicial syllogism, mediate deductive inferences, monotonic logic, non-monotonic logic

Introduction

The application of logic in law has long been essential for ensuring clarity, structure, and consistency in legal reasoning. Among the most notable examples of this logical application is the judicial syllogism, a framework that enables the application of general legal principles to particular cases in all areas of law (Tasev et al., 2020). Alongside the legislative syllogism, it forms part of the broader conceptual structure known as the legal syllogism. Both rely on the structure of the mediate deductive

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inference, a form of reasoning that progresses from general premises to specific conclusions, involving a middle term that bridges the premises to derive the conclusion, with certain particularities since both the major premise and the conclusion are normative, prescriptive propositions. For instance, a classical example of a judicial syllogism in a linguistic form is structured as such:

> Major premise: "All citizens are subject to the law."
> Minor premise: "*X is a citizen.*"
> Conclusion: "*Therefore X is subject to the law.*"

In this example, the term "citizen" serves as the link between the premises and the conclusion and in a judiciary process, the logical structure underlying this inference ensures that the general legal rule applies consistently to specific factual circumstances.

However, despite its historical acceptance and theoretical clarity, the classical judicial syllogism shows significant limitations, since it is grounded in classical deductive reasoning which makes it inherently static, meaning that once the conclusion was derived it cannot be changed. This is useful, however, in a final or at least in a very advanced moment in the judicial process where all matters of law and facts have already been clarified. But before reaching such a static point in the judicial process, the monotonic judicial syllogism does not account for the dynamic and often unpredictable course of legal procedures which imply addressing *quaestio juris*, questions of law, and *quaestio facti*, questions of fact (Codrea, 2023a). These two issues frequently introduce new information or nuances that may alter the premises, generate new syllogisms or chain of such deductive inferences, thereby challenging the initial conclusion, in opposition to classical deductive reasoning, grounded in monotonic logic which remains rigid, as its conclusions stay unchanged even when the initial premises are modified, or new premises emerge. Therefore, the limitations of monotonic logic become especially evident when it fails to incorporate in a logical manner the changes introduced by evolving facts or legal interpretations during a judicial process. Thus, a more adaptable approach involves integrating non-monotonic logic into the judicial syllogism itself, allowing conclusions to be revisited and revised when new information becomes available altering the premises, in order to address the dynamic aspects of legal adjudication while preserving the overarching deductive structure of the judicial syllogism which itself offers the logical structure for the judicial process (Codrea, 2023c).

Before analyzing the place where non-monotonic logic may be integrated in the broader framework of the judicial syllogism, a previous analysis of mediate deductive inferences in logic is required, since the judicial syllogism is a part of it, and its functioning follows a similar pattern. The analysis will continue with the legal syllogism focusing on the classical judicial syllogism, leaving aside the logical problem of Jörgensen dilemma (Jørgensen, 1937) regarding the logical status of the propositions composing the judicial syllogism. At this point, the analysis will focus on identifying specific stages in the construction of the judicial syllogism where non-

monotonic reasoning could be relevant, with the purpose of reconciling the deductive precision of classical logic with the practical demands of legal adjudication.

1. Logical analysis of deductive inferences or reasoning

Reasoning, a concept that currently holds a rather psychological meaning, while the logical term is that of inference, is generated by inter-propositional relations, except for independence relation. Every inference is grounded on a logical principle; however, not all logical principles constitute inferences – only those that assume the form of implication or equivalence. In this sense, neither the logical principle of non-contradiction nor the principle of the excluded middle represents implications (Botezatu, 1997; Kreeft, 2010; Lee, 2017).

In modern logic, the rules of inference governing inter-propositional relations operate by transforming statements independently of their semantic content, without regard to their meaning. As a technique for deriving one proposition from other propositions, the inference represents both a logical operation and a logical form, relying on a set of rules that govern a specific language (Botezatu, 1997; Kreeft, 2010).

Structurally, an inference consists of premises, which are the given propositions, and the conclusion, which is the new proposition derived from the premises. A set of propositions constitutes an inference if it meets the following rules (Botezatu, 1997; Lee, 2017):
1. There are given propositions – the premises;
2. From the given propositions, a new proposition follows in relation to the first – the conclusion;
3. The premises represent either a sufficient or a necessary condition for the conclusion, in the sense that nothing else is required for deriving the conclusion;
4. The conclusion represents either a necessary or sufficient consequence of the premises, in the sense that if the premises are given, the conclusion must necessarily follow.

Thus, the connection between premises and conclusion adheres to the principle of sufficient reason, which underlies all reasoning or inferences. Although the premises may be multiple, the conclusion is unique. An inference is valid or correct if its premises are true. A valid inference with true premises is called conclusive, the only one that guarantees true, demonstrated conclusions, while a non-conclusive inference may randomly yield true conclusions, but these remain undemonstrated.

Traditional logic classifies inferences according to several criteria (Botezatu, 1997; Scambler, 2020): by the direction of inference from general to particular, inferences are deductive or inductive; by the number of premises, deductive inferences are classified into immediate and mediate. If we focus on mediate

deductive inferences, they are classified by the type of premises into categorical, hypothetical and disjunctive.

1.1. Hypothetical deductive inferences

Hypothetical Deductive Inferences are those inferences composed of hypothetical propositions. Depending on the prevalence of hypothetical propositions, hypothetical inferences can be classified as pure or mixed (Botezatu, 1997; Codrea, 2021; Dumitriu, 1969).

The pure hypothetical inference consists exclusively of hypothetical propositions and can be expressed in the following forms:

"If p then q"	$p \supset q$
"If q then r"	$q \supset r$
"Therefore, if p then r"	$\therefore p \supset r$

This type of inference reflects the principle that the consequence of a consequence is the consequence of the condition, with all propositions based on a relationship of conditioning (implication). In modern logic, the pure hypothetical inference is called a hypothetical syllogism.

The mixed hypothetical inference or hypothetical-categorical inference contains only the first premise as a hypothetical proposition, while the second premise and the conclusion are categorical propositions. Mixed hypothetical inference can express multiple relationships of conditioning.

If the premise is a standard hypothetical proposition, it expresses a relationship of sufficient conditioning and can take the following forms:

Modus Ponens (affirming the antecedent):

"If p then q"	$p \supset q$
"p is true"	p
"Therefore, q is true"	$\therefore q$

Modus Tollens (denying the consequent):

"If p then q"	$p \supset q$
"q is false"	$\neg q$
"Therefore, p is false"	$\therefore \neg p$

If the premise is an exclusive hypothetical proposition, it expresses a relationship of sufficient and necessary conditioning (if p then q and if q then p), where the relationship between p and q is expressed as $p \equiv q$. This can take the following four forms, two for each mode:

Modus Ponens (from condition/ from consequence):

$p \equiv q$	$p \equiv q$
p	q
$\therefore q$	$\therefore p$

Modus Tollens (from condition/ from consequence):

$p \equiv q$	$p \equiv q$

$$\neg\mathbf{p} \qquad\qquad\qquad \neg\mathbf{q}$$
$$\therefore \neg\mathbf{q} \qquad\qquad\qquad \therefore \neg\mathbf{p}$$

This type of inference expresses that the truth of the consequence implies the truth of the condition, and the falsity of the condition implies the falsity of the consequence.

If the premise is a non-exclusive hypothetical proposition, the result consists of paralogisms, which are false reasoning or invalid inferences.

1.2. Disjunctive deductive inferences

Disjunctive deductive inferences are composed of disjunctive propositions whose predicates are opposed, being governed by the principle of non-contradiction and the principle of the excluded middle (Codrea, 2018; Kreeft, 2010; Lee, 2017).

The disjunctive-categorical inferences, also called mixed inferences, assume that only the major premise is a disjunctive proposition, while the minor premise and the conclusion are categorical (Botezatu, 1997).

When propositions are incompatible, the relationship is subject to the principle of non-contradiction, which prohibits the propositions from being true simultaneously, and the disjunction must be exclusive, though not necessarily complete. Therefore, the truth of one proposition implies the falsity of the other, and the inferences are based on the relationship of incompatibility. These inferences occur only in *modus ponendo tollens*, where the conclusion denies by affirming in the premises, and they can be expressed in the following forms:

$$\neg(\mathbf{p} \wedge \mathbf{q}) \qquad\qquad \mathbf{p} / \mathbf{q}$$
$$\mathbf{p} \qquad\qquad\qquad \mathbf{p}$$
$$\therefore \neg\mathbf{q} \qquad\qquad\qquad \therefore \neg\mathbf{q}$$

and

$$\neg(\mathbf{p} \wedge \mathbf{q}) \qquad\qquad \mathbf{p} / \mathbf{q}$$
$$\mathbf{q} \qquad\qquad\qquad \mathbf{q}$$
$$\therefore \neg\mathbf{p} \qquad\qquad\qquad \therefore \neg\mathbf{p}$$

When there are only two propositions that are not exclusive, but the disjunction is complete, the relationship is subject to the principle of the excluded middle. The propositions cannot be false simultaneously, so the falsity of one implies the truth of the other. These inferences occur only *in modus tollendo-ponens*, as they affirm the conclusion by negating one of the premises, and they can be expressed in the following forms:

$$\mathbf{p} \vee/\wedge \mathbf{q} \qquad\qquad \mathbf{p} \vee \mathbf{q}$$
$$\neg\mathbf{p} \qquad\qquad\qquad \neg\mathbf{p}$$
$$\therefore \mathbf{q} \qquad\qquad\qquad \therefore \mathbf{q}$$

and

$$\mathbf{p} \vee/\wedge \mathbf{q} \qquad\qquad \mathbf{p} \vee \mathbf{q}$$
$$\neg\mathbf{q} \qquad\qquad\qquad \neg\mathbf{q}$$

$$∴ \mathbf{p} \qquad\qquad ∴ \mathbf{p}$$

When the disjunction is exclusive, both modes are possible:

Modus ponendo-tollens:

$$\mathbf{p} \not\equiv \mathbf{q} \qquad\qquad \mathbf{p} \not\equiv \mathbf{q}$$
$$\mathbf{p} \qquad\qquad\qquad \mathbf{q}$$
$$∴ \neg\mathbf{q} \qquad\qquad ∴ \neg\mathbf{p}$$

Modus tollendo-ponens:

$$\mathbf{p} \not\equiv \mathbf{q} \qquad\qquad \mathbf{p} \not\equiv \mathbf{q}$$
$$\neg\mathbf{p} \qquad\qquad\qquad \neg\mathbf{q}$$
$$∴ \mathbf{q} \qquad\qquad ∴ \mathbf{p}$$

Disjunctive-hypothetical inferences are also called dilemmas (Codrea, 2023b) or trilemmas, tetralemmas, polylemmas, and assume that through hypothetical propositions the disjunction is transferred from premises to other propositions or to a single proposition in the conclusion (Botezatu, 1997; Espino & Byrne, 2013). The structure of the dilemma consists of a major premise composed of two hypothetical propositions, a minor premise composed of a disjunctive proposition and a conclusion.

A dilemma is considered constructive when the minor premise affirms both antecedents of the major premise. If the resulting conclusion follows the same sequence, the dilemma is classified as simple; however, if different sequences lead to distinct conclusions, it is classified as complex (Botezatu, 1997; Juhos et al., 2012).

The simple constructive dilemma assumes the following form:

$\mathbf{p} ⊃ \mathbf{q}$, "If *p* then *q*"
$\mathbf{r} ⊃ \mathbf{q}$, "If *r* then *q*"
$\mathbf{p} ∨ \mathbf{r}$, "*p* or/& *r*"
$∴ \mathbf{q}$, "Therefore *q*"

The complex constructive dilemma assumes the following form:

$\mathbf{p} ⊃ \mathbf{q}$, "If *p* then *q*"
$\mathbf{r} ⊃ \mathbf{s}$, "If *r* then *s*"
$\mathbf{p} ∨ \mathbf{r}$, "*p* or/& *r*"
$∴ \mathbf{q} ∨ \mathbf{s}$, "Therefore *q* or/& *s*"

A dilemma is classified as destructive when the minor premise negates both sequences of the major premise. If the conclusion follows the same sequence, it is termed a simple dilemma; however, if different sequences lead to distinct conclusions, it is termed a complex dilemma.

The simple destructive dilemma assumes the following form:

$\mathbf{p} ⊃ \mathbf{q}$, "If *p* then *q*"
$\mathbf{p} ⊃ \mathbf{r}$, "If *p* then *r*"
$\neg\mathbf{q} ∨ \neg\mathbf{r}$, "non-*q* or/& non-*r*"
$∴ \neg\mathbf{p}$, "Therefore non-*p*"

The complex destructive dilemma assumes the following form:

p ⊃ **q**, "If *p* then *q*"
r ⊃ **s**, "If *r* then *s*"
¬**q** ∨ ¬**s**, "non-*q* or/& non-*s*"
∴ ¬**p** ∨ ¬**r**, "Therefore non-*p* or/& non-*r*"

1.3. Categorical deductive inferences

Categorical deductive inferences consist of categorical propositions. In a broad sense, any mediated, hypothetical, disjunctive, or categorical inference can be termed a syllogism; however, in a narrower sense, the term refers specifically to a deductive mediated inference. The initial theorization of the syllogism is entirely attributed to Aristotle, who defines it as

> a speech in which, if something has been given, something other than the given follows of necessity from what has been given. I understand by the expression: from what has been given, that from this always follows a consequence, and by this latter expression, that no other term from outside is needed to make the consequence necessary (Aristotle, 1958).

From Aristotle's definition of the syllogism, it follows that the premises, which Aristotle calls *protas*, must constitute a sufficient condition for obtaining the conclusion, and the conclusion must represent a necessary consequence of the premises. Aristotle also identifies the components of the syllogism, which consists of three terms: the major term, or the first extreme; the minor term, or the last extreme; and the middle term (Dumitriu, 1969).

By incorporating the terms within both their intensional and extensional dimensions, the principle underlying the syllogism has been expressed through various formulations:
- three terms that are successively included one in the sphere of the other;
- what is attributed to all is also attributed to some and one, and what is not attributed to any is also denied to any individual or subgroup within that set or category (*dictum de omni et nullo*);
- in Leibniz's formulation (Leibniz, 1875), the includent of the includent is the includent of the included *(includens includentis est includens inclusi)*;
- either the genus of the genus is the genus of the species, or the species of the species is the species of the genus;
- in modern logic, the syllogism is based on the property of transitivity of the inclusion relation of sets, in the form: $(z \subset y) \wedge (y \subset x) \supset (z \subset x)$ (Botezatu, 1997; Dumitriu, 1969).

Structurally, the syllogism is highlighted by the following (Senturk & Oner, 2019):
1. it contains three terms, called major, middle, minor, according to the size of the sphere, where the major and minor are called extremes;
2. the middle term appears in both premises and does not appear in the conclusion, having the role of ensuring the connection between the extremes (**M**);
3. the extreme terms appear separately in each premise and together in the conclusion;
4. the major term is the predicate of the conclusion (**P**), and the minor term is the subject of the conclusion (**S**);
5. the syllogism contains three propositions: two premises and a conclusion. The premise that contains the major term is called major, and the one that contains the minor term is called minor.

> All **M** are **P**.
> All **S** are **M**.
> ∴ All **S** are **P**.

The syllogism is governed by the following rules (Botezatu, 1997; Dyckhoff, 2019):
1. it contains three terms;
2. the conclusion does not contain the middle term;
3. a term cannot be distributed in the conclusion if it has not been distributed in the premises;
4. the middle term must be distributed in at least one of the premises;
5. a negative conclusion cannot result from two affirmative premises;
6. a conclusion cannot be derived from two negative premises;
7. if a premise is negative, the conclusion is negative;
8. if a premise is particular, the conclusion is particular;
9. a conclusion cannot be derived from two particular premises.

Depending on the position in the premises of the middle term **M**, which can be a subject **S** or a predicate **P**, 4 syllogistic figures emerge, each with its own rules. Within each figure, several modes or forms of syllogism are possible depending on the quality and quantity of the premises and the conclusion, which, in principle, can have the 4 forms (**A**) *All S are P*, **SaP**, (**E**) *No S is P*, **SeP**, (**I**) *Some S are P*, **SiP**, (**O**) *Some S are not P*, **SoP**. Considering also that the syllogism involves 3 propositions, 256 possible syllogistic modes result, of which only 24 are valid, 6 for each figure. In modern logic, syllogistic belongs to the logic of monadic predicates or to the logic of sets or classes. Of the 24 modes, only 15 have been validated. All modes in which particular conclusions in **I** and **O** are derived from universal premises in **A** and **E** are not valid, due to the existential significance of the particular propositions. In order for these modes to be validated, it is necessary to introduce a premise regarding the existence of objects in one of the sets **S**, **P** or **M**, a premise that in traditional logic was implicit (Weingartner, 2017).

Syllogisms can appear in abbreviated or compound forms. The abbreviated form is called an enthymeme or elliptical syllogism, and the compound form is called a polysyllogism.

The enthymeme was identified by Aristotle and represents the elliptical syllogism in which one of the three propositions, the major premise, the minor premise, or the conclusion, is not expressed in the syllogism, being implied: the first order enthymeme does not express the major premise, the second order enthymeme does not express the minor premise, and the third order enthymeme does not express the conclusion (Botezatu, 1997; Dyckhoff, 2019).

A polysyllogism is an inference composed of several syllogisms in which the conclusion of the first syllogism, called the prosyllogism, is the premise of the next syllogism, called the episyllogism. Depending on the role of the conclusion of the prosyllogism, which can be a major premise or a minor premise for the episyllogism, the polysyllogism can be progressive or regressive. If these two forms are simplified by suppressing the intermediate conclusions, the result is sorites. Sorites can be either Goclenian, if it results from the simplification of the progressive polysyllogism, or Aristotelian, if it results from the simplification of the regressive polysyllogism (Botezatu, 1997).

The progressive polysyllogism implies that the conclusion of the prosyllogism becomes the major premise for the episyllogism and has the following form:

All **M** are **P**	**MaP**
All **N** are **M**	**NaM**
∴ All **N** are **P**	∴ **NaP**
All **S** are **N**	**SaN**
∴ All **S** are **P**	∴ **SaP**

The Goclenian sorites:

M is **P**	**MaP**
N is **M**	**NaM**
S is **N**	**SaN**
∴ **S** is **P**	∴ **SaP**

Rules of Goclenian sorites:
  1. Only one premise can be negative: the first.
  2. Only one premise can be particular: the last.

The regressive polysyllogism implies that the conclusion of the pro-syllogism becomes the minor premise of the epi-syllogism

All **S** are **N**	**SaN**
All **N** are **M**	**NaM**
∴ All **S** are **M**	∴ **SaM**
All **M** are **P**	**MaP**
∴ All **S** are **P**	∴ **SaP**

The Aristotelian sorites:

S is **N**	**SaN**

$$\begin{array}{ll} \textbf{N} \text{ is } \textbf{M} & \textbf{NaM} \\ \textbf{M} \text{ is } \textbf{P} & \textbf{MaP} \\ \therefore \textbf{S} \text{ is } \textbf{P} & \therefore \textbf{SaP} \end{array}$$

The rules of Aristotelian sorites:
1. Only one premise can be negative: the last.
2. Only one premise can be particular: the first.

2. The judicial syllogism

In a study dedicated to the application of the principle of sufficient reason in legal thinking, Eugeniu Speranția observes: "We find ourselves (...) always in the realm of legal thinking as long as we explain one imperative formula by another, more general one. This is the true process of justifying a legal norm. A judicial sentence is justified by the legal text on which it is rationally based, being merely its concrete application. A specific legal text appears justified within a legal order when it translates into a particular form an imperative of a more general character found among the provisions of an organic law, when it logically derives from a principle established in the constitution, from a universally accepted norm of the dominant mentality of the time and social group, or from a necessary and a priori exigency of human thought" (Speranția, 1940). In this excerpt it is illustrated the application of the principle of sufficient reason in law through the legal syllogism, which is classified into two types: the judicial syllogism, corresponding to the moment of applying the legal norm to a particular case, and the legislative syllogism, which pertains to the process of drafting a legal norm.

The judicial syllogism is the inference through which a factual situation in reality is framed within a legal norm by a judicial decision. In its simplified form, it consists of:
1. Major premise: The legal norm in the form "*If x, then y must follow*."
2. Minor premise: The factual situation in reality "*x exists*."
3. Conclusion: The judicial decision in the form "*y must follow*."

Therefore, this is a type of mediate deductive inference, as Aarnio noticed "The structure of the legal syllogism allows legal norms to function as major premises, enabling deductive reasoning to produce enforceable conclusions" (Aarnio, 1987). Judicial syllogism allows the application of general legal rules to particular facts of a case, forming a logical bridge between law and decision (MacCormick, 1978). The role of this inference is to ensure that decisions follow logically from established legal rules and facts, providing a proper structure, a logical framework for consistency in judicial reasoning (MacCormick, 1978).

However, although considered the fundamental logical structure of legal reasoning, its application always requires interpretation, as the meaning of legal norms is not self-evident (Alexy, 1989). Precisely the process of interpretation of both facts and legal norms makes the monotonic inference problematic and requires

the inclusion of non-monotonic logic to account for the dynamic of answering *quaestio juris* and *quaestio facti* (Bochman, 2008). As indispensable as it is considered for ensuring that judicial decisions appear rational and logically justified, "its simplicity belies the interpretative challenges it entails" (Alexy, 1989).

If we consider a more detailed formulation of a legal norm prohibiting any ($\forall$) legal subject (***x***) from engaging in a specific conduct (***C***) under a certain sanction (***S***), the judicial syllogism will take the following form:

1. Major premise: The legal norm in the form $\forall x(\neg C(x) \rightarrow S(x))$**,** meaning "$\forall$ *x*, *if* ¬*C(x)*, *then it is required S(x)*", for any legal subject ***x***, if ***x*** does not have the conduct ***C***, ***x*** must be sanctioned with ***S***.

2. Minor premise: The factual situation from reality with a particular character in the form $\exists \neg C1(x1)$, meaning that the legal subject ***x1*** did not have the conduct ***C1***.

3. Conclusion: The court decision in the form ***S(x1)***, meaning "*It is required S(x1)*", the legal subject ***x1*** must be sanctioned with ***S***.

It can be observed that a judicial syllogism composed exclusively of the major and minor premises does not allow for the formulation of the conclusion, as there is no common middle term in the two premises:

- The major premise contains the major term $\forall x(\neg C(x) \rightarrow S(x))$, and a middle term ¬***C(x)***.
- The minor premise contains the minor term ***x1*** and another middle term "$\exists$ ¬*C1(x1)*".

To establish a judicial syllogism that allows for deriving the conclusion "*It is required S(x1)*" the relationship "*¬C1(x1)* $\equiv$ *¬C(x)*" must first be established. This subsequently requires the determination of two inclusion relationships: "***x1*** $\subset$ ***x***" and "***C1*** $\subset$ ***C***".

Thus, the judicial syllogism that allows for deriving the conclusion must be completed with a third and a fourth premise:

1. Major premise: The legal norm in the form $\forall x(\neg C(x) \rightarrow S(x))$, meaning "$\forall$ *x*, *if* ¬*C(x)*, *then it is required S(x)*", for any legal subject ***x***, if ***x*** does not exhibit conduct ***C***, then ***x*** must be sanctioned with ***S***.

2. Minor premise: The factual situation from reality with a particular character in the form "$\exists$ ¬*C1(x1)*", meaning that the legal subject ***x1*** did not exhibit conduct ***C1***.

3. Third premise: "***x1*** $\subset$ ***x***", meaning that the legal subject ***x1*** is included in the category of legal subjects ***X***, making the legal norm in the major premise applicable to ***x1***.

4. Fourth premise: "***C1*** $\subset$ ***C***", meaning that the specific conduct ***C1*** constituting the particular factual situation is included in the conduct ***C*** sanctioned by the legal norm.

5. Conclusion: The court decision in the form "*It is required S(x1)*", meaning that the legal subject ***x1*** must be sanctioned with ***S***.

Regardless of the specific formulation of the judicial syllogism, any statement assumed by the major premise, which always consists of a legal norm, does not possess an assertive, cognitive, descriptive, or declarative nature regarding *how facts in reality actually are*. Instead, it has a normative, prescriptive, and imperative nature, containing commands and orders that refer to *how facts in reality ought to be*. This particularity of normative statements poses certain logical challenges to judicial syllogism, analyzed by Jørgensen and established through the dilemma that has his name (Jørgensen, 1937). Without elaborating here on the logical implications of Jørgensen's Dilemma, informally, primarily through the validation of legal doctrine and judicial practice, it is generally accepted that the judicial syllogism is a case of deduction from the general to the particular that is intuitively valid, as Aarnio states: "In legal dogmatics, the syllogism operates as a link between norms and facts, transforming abstract legal provisions into concrete judgments" (Aarnio, 1987). Normative statements can be assimilated to descriptive ones, making it possible to construct a syllogism either exclusively from descriptive premises, exclusively from normative premises, or from a combination of normative and descriptive premises, as it is the case with the judicial syllogism (Volpe, 1999). In this latter case, the judicial syllogism assumes that if one of the premises is normative, then the conclusion must also be normative (Coyle, 2004).

Following the formal structure of the judicial syllogism, it can be observed that the entire judicial process consists merely of various methods of grounding, justifying, proving, or grounding each of the premises. The court's role is, on the one hand, to address the *quaestio juris* by identifying the major premise, which is provided by the valid legal norm applicable to the case under review, and on the other hand, to address the *quaestio facti* by identifying the minor premise, which is determined by the factual circumstances in reality. The qualification of the minor premise also responds to the *quaestio juris* by relating it to the *quaestio facti*, so as to logically subsume the particular factual situation under the general legal norm subject to judgment.

For the decision rendered in the case to be grounded in both law and fact, the premises of the inference, whose conclusion is the decision itself, must also be valid. The invalid grounding of the premises of the judicial syllogism leads to the formulation of an unsubstantiated conclusion, which, in turn, provides the legal basis for lodging an appeal against the respective court decision.

3. The grounding of judicial syllogism and non-monotonic logic

Traditionally, the grounding of the major premise relates to the answer given to the *quaestio juris*, while the grounding of the minor premise relates to the answer given to the *quaestio facti*. However, although *quaestio juris* and *quaestio facti* are presented as distinct, the answers provided to these two questions complement each other in a successive manner. While the grounding of the major premise is

exclusively connected to *quaestio juris*, the entire operation of qualifying the minor premise through establishing the validity of the third premise $x1 \subset x$ and the fourth premise $C1 \subset C$, though originating from the answer to *quaestio facti*, contributes to the answer to the *quaestio juris*.

Similarly, the grounding of the minor premise, concerning the specific factual state in reality, namely the existence of $\neg C1(x1)$, relates to the *quaestio facti*, but also contributes to qualification by establishing the validity of the third premise $x1 \subset x$ and the fourth premise $C1 \subset C$. Consequently, neither logically nor legally is it relevant to analyze separately the answers given to the *quaestio juris* and the *quaestio facti*. Instead, the focus should be on the distinct logical-legal grounding of the two premises of the judicial syllogism, where factual and legal aspects are in a relationship of mutual interdependence.

3.1. The grounding of the major premise

Regarding the grounding of the major premise, this always involves a legal norm. A legal norm derives its validity, is considered valid, or is grounded by reference to another legal norm, in a variable succession, up to the supreme legal norm, which is typically the Constitution in a legal system. Any judicial syllogism is therefore based on a more or less complex structure of logically and hierarchically linked legal norms, but this structure itself is justified in its entirety by an ultimate legal imperative that possesses such authority as to serve as the pivot of the entire legal life and the sufficient reason for the whole legal system (Codrea, 2023c).

Without delving into the details regarding the logical structure of the legal normative system or the process of logical derivation of legal norms, this ultimate norm, upon which the entire legal system is grounded and referred to by Hans Kelsen as the *Grundnorm* or basic norm, (Kelsen, 1960) emerges as a transcendental-logical necessity for the existence of any system of positive law and, subsequently, for the grounding of any other legal norm (Codrea, 2020; Ziembinski, 1970).

3.2. The grounding of the minor premise

In order to ground the minor premise "$\exists \neg C1(x1)$", an analysis of the judicial syllogism is necessary. The function of the minor premise, which represents the factual situation in reality with a particular character, or the specific case under judgment in the form "$\exists \neg C1(x1)$", is to subsume the minor term $x1$ under the middle term $\neg C(x)$. This allows for the application of the general rule from the major premise $\forall x(\neg C(x) \rightarrow S(x))$ in the conclusion. The court must assess whether the legal subject $x1$ falls under the legal norm by establishing that $x1 \subset x$ and that $x1$ engaged in the conduct $C1 \subset C$, which is sanctioned by the legal norm through S.

While fulfilling this role in the judicial syllogism, the grounding of the minor premise involves determining a factual situation in reality: "$\exists \neg C1(x1)$". If the major

premise consists of a legal norm and contains a normative statement about *how factual situations in reality should be*, the minor premise consists of a concrete factual situation from reality and contains a descriptive statement about *how reality is*. Therefore, the method for establishing the minor premise is distinct from that of the major premise, involving operations of describing and interpreting a factual situation that can be ascertained and reconstructed through various means of evidence. Depending on the subject matter or nature of the case, specific procedural legal norms regulate what constitute evidence, admissibility, administration, and evaluation, as well as the means of evidence and evidentiary procedures. In the non-monotonic logic developed by Reiter, all these elements regarding the factual situation which can lead to overriding the minor premise is noted with *D*, from "defense" and *E* from "exceptions" (Reiter, 1980).

3.3. Qualifying the minor premise

To construct the judicial syllogism from the major and minor premises, allowing for the derivation of a conclusion in the form of a judgment, the court must qualify the minor premise. This requires evaluating the validity of the third and fourth premises.

The determination of these two premises relies on both factual and legal elements, being encompassed in the answers to the *quaestio juris* and *quaestio facti*. It involves determining the validity of the following relations:

1. Third premise: $x1 \subset x$, meaning the legal subject $x1$:
    1. is included in the category of legal subjects x, making the legal norm from the major premise applicable to it;
    2. meets the conditions required by law to bear legal responsibility.
2. Fourth premise: $C1 \subset C$, meaning the conduct $\neg C1$ of $x1$, which exists as a concrete factual situation, is included in the conduct $\neg C$ that is sanctioned by the legal norm.

The judicial operation through which the court establishes the validity of these relationships is related to the legal qualification process, specifically the classification of the minor premise "$\exists \neg C1(x1)$" – the concrete situation – under a legal norm. In addition to the validity of the legal norm, which concerns the determination of the major premise, the applicability of the legal norm to the factual situation under judgment constitutes another aspect of the *quaestio juris*. In this respect, the court is tasked with assessing whether the legal subject $x1$, judged on the factual situation provided by the valid minor premise in the form "$\exists \neg C1(x1)$", belongs to the category of legal subjects addressed by the norm $\forall x(\neg C(x) \rightarrow S(x))$. Furthermore, the court must determine whether the conditions required by law for engaging legal liability are met.

Therefore, the answer to the *quaestio juris* does not stop at validating the major premise. Instead, it also contributes, together with the answer to the *quaestio*

facti – which involves determining that "$\exists \neg C1(x1)$" – to establishing the inclusion relationship $x1 \subset x$. This inclusion relationship is essential for constructing the judicial syllogism.

Similarly, even if it has been established through evidence that "$\exists\ C1(x1)$", the court must evaluate the validity of the relationship $C1 \subset C$. This means determining whether the conduct that occurred in reality, $\neg C1$, meets the conditions imposed by law to engage legal liability based on the major premise $\forall x(\neg C(x) \rightarrow S(x))$. The invalidity of the relationship $C1 \subset C$ can alter the initial answer to the *quaestio juris*, leading to a change in the major premise and the generation of a different judicial syllogism.

Determining the third and fourth premises is the point where non-monotonic logic can play a major role, since it is in those two relations $x1 \subset x$ and $C1 \subset C$ that defeasibility and exceptions are introduced, and conclusions can change based on new evidence or context (Halpern, 2003). For example, the Default Logic elaborated by Reiter in 1980 with its additional formalization can be valuably integrated in the judicial syllogism (Reiter, 1980). All those elements described above regarding the factual situation which can sufficiently amount to the invalidation of $(x1 \subset x)$ or $(C1 \subset C)$ or both, and thus, to the invalidation of the minor premise, are noted with $D(x)$, from "defense" and $E(x)$ from "exceptions" which are presented during the judicial process. Therefore, the logical process would take this form: $((D(x1) \lor E(x1)) \rightarrow \neg((x1 \subset x)\ \lor (C1 \subset C))$.

3.4. Non-monotonic logical steps for developing the judicial syllogism

As shown above, only by determining the third and fourth premises is it possible to construct the judicial syllogism and derive the conclusion "*It is required S(x1)*". Logically, the following steps are necessary for developing the judicial syllogism:

1. Determining the major premise, defined by the legal norm in the form $\forall x(\neg C(x) \rightarrow S(x))$. This means that for any legal subject x, if it does not have the conduct C, then x must be sanctioned by S. It includes:
   - The major term: $\forall x$ *it is required S(x)*
   - The middle term: $\neg C(x)$.
2. Determining the minor premise, defined by the factual situation in reality with a particular character in the form "$\exists \neg C1(x1)$". It includes:
   - The minor term: $x1$
   - The middle term: $\neg C1(x1)$.
3. Since the middle term in the major premise, $\neg C(x)$, and the middle term in the minor premise, $\neg C1(x1)$, are not identical, the court must evaluate the validity of the following premises to formulate the conclusion:

1. $X1 \subset X$: This means the legal subject $x1$ is included in the category of legal subjects x to whom the legal norm applies and meets the legal conditions for bearing legal responsibility, if $D(x1) \lor E(x1)$ is insufficient

2. $C1 \subset C$: This means the conduct $C1$, which constitutes the concrete factual situation, is included in the conduct C sanctioned in the major premise, if $D(x1) \land E(x1)$ is insufficient

3. Based on 1) and 2), if $D(x1) \lor E(x1)$ is insufficient, the middle term becomes equivalent: $\neg C1(x1) \equiv \neg C(x)$.

4. *Deriving the conclusion and issuing the judgment:* "*it is required S(x1)*".
   - The conclusion does not contain the middle term $\neg C1(x1) \equiv \neg C(x)$, which appears in both premises.
   - The conclusion contains the minor term $X1$.
   - The conclusion contains the major term "*it is required S(x)*", and since $x1 \subset x$, it follows that *it is required S(x)* $\equiv$ *it is required S(x1)*, as the predicate of the conclusion.

   Thus, a function of the third premise ($X1 \subset X$) and the fourth premise ($C1 \subset C$) is to allow for the establishment of the middle term through the equivalence $\neg C1(x1) \equiv \neg C(x)$. This equivalence forms the judicial syllogism and makes it possible to derive the conclusion "*it is required S(x1)*". Otherwise, if in the third or forth premise there are issues establishing the inclusions through sufficient additional information $((D(x1) \lor E(x1)))$, then the conclusion should change to $\neg S(x1)$, "*it is not required S(x1)*".

5. This judicial syllogism is constructed on a specific norm, $N1: \forall x(\neg C(x) \rightarrow S(x))$. If in the previous steps $((D(x1) \lor E(x1)) \rightarrow \neg((x1 \subset x) \lor (C1 \subset C))$ then it follows that "*it is not required S(x1)*". However, non-monotonic logic allows for changing the major premise given new information, which is added through $(D(x1) \lor E(x1))$. Therefore, if there is a different legal norm $Nn: \forall y(\neg Cn(y) \rightarrow Sn(y))$ it follows that $(N1 \land (D(x1) \lor E(x1))) \rightarrow Nn$ and a new judicial syllogism can be constructed on the same pattern with Nn as a major premise.

   To ellaborate on Reiter's Default Logic, considering the new information $D(x)$ and $E(x)$:

- $((D(x1) \lor E(x1)) \rightarrow \neg((x1 \subset x) \lor (C1 \subset C)) \rightarrow \neg C1(x1) \not\equiv \neg C(x) \rightarrow \neg N1 \rightarrow \neg S(x1)$
- $((D(x1) \lor E(x1)) \rightarrow ((x1 \subset y) \land (C1 \subset Cn)) \rightarrow \neg C1(x1) \equiv \neg Cn(y) \rightarrow Nn \rightarrow Sn(x1)$

Conclusions

The judicial syllogism remains a fundamental component of legal reasoning, offering a logical structure to bridge general legal principles with specific cases. However, its foundation in classical monotonic logic limits its capacity to reflect the

dynamic and evolving nature of judicial processes and this article has proposed integrating non-monotonic logic to address these limitations, allowing the judicial syllogism to adapt to new facts, defenses, and interpretations without losing its deductive coherence. By incorporating frameworks such as defeasible reasoning and Reiter's default logic, the judicial syllogism can better mirror the realities of legal decision-making, where conclusions must often be revisited in light of emerging information and this adaptability not only aligns legal reasoning with practical judicial needs but also ensures that the reasoning process remains logical and justifiable. The practical applications of this theoretical model could span from developing computational tools to conducting case studies to assess its functionality in real-world settings which could subsequently contribute to developing AI applications.

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