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Population ageing and sustainable fiscal policy in Czechia

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Abstract

The substantial ageing impact projected by the Eurostat motivates this research for the case of the Czech Republic. The study assists policymakers by analysing fiscal-policy measures to stabilize ageing impact on the income and the well-being; and motivates fiscal authorities to utilize an extended version of the Czech Ministry of Finance model for the demographic agenda. The examined fiscal measures consist of postponing retirement, increasing pensions, and reducing social-security payments. The simulation outcome reveals reducing social-security payments as the only fiscal policy that would maintain both labour income and well-being unaltered in the presence of ageing. The study continues by proposing a policy mix to mitigate the subsequent government deficit. The policy mix consists of increasing the VAT tax rate and decreasing pensions and other transfers. In conclusion, the reduction of the social security payments financed by the suggested policy mix would support individuals' responsibility for their future income while motivating them towards higher productivity during their younger years.

Keywords: ageing, DSGE, well-being, Czech Republic

Introduction

The problem of ageing population is at the forefront of the ongoing policy debates in many EU member states. The current pension scheme appears fiscally unsustainable for the projected demographic shifts in most European countries (see Langenus and Geert, 2006) and requires policy makers to search for new policy measures.

The severity of this problem in the Czech Republic motivates the focus on this economy. Specifically, the research will evaluate the capacity of different fiscal policy measures to mitigate the ageing effect on Czech labour income and well-being

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deterioration. The applied model is an extended Dynamic Stochastic General Equilibrium (DSGE) model from the Czech Ministry of Finance (Aliyev *et al.*, 2014).

The Eurostat demographic projections predict that the Czech population will face a gloomy future with the rise in the old-age dependency ratio. The forecast expects the current share of people aged over sixty years to grow from 26 percent to around 36 percent in 2050. Simultaneously, the fertility rate should only slightly rise from 1.7 in 2018 to 1.8 percent in 2050 (see Eurostat projections[1]). Figure 1 in the Appendix illustrates the dynamics for individual age cohorts. This ageing trend will affect everyone. Prolonged life expectancy will enable individuals to work longer, which differentiates the long-term saving/consumption choices for the current generation from their parents' decision-making. At the same time, the government is short on finances for the increasing number of eligible retirees. Therefore, the Czech fiscal authorities rush to create an economically feasible pension reform.

The pension reform needs to be fiscally sustainable while minimizing the negative ageing impact on the Czech population, such as smaller disposable income and well-being. While the evaluation of the income effects is more common (see Babecky and Dybczak, 2009); Fleurbaey (2009), Benjamin *et al.* (2014) stressed the importance of also measuring well-being as an appropriate criterion. This study analyses both the income and the well-being criterion.

This research is also unique in searching for the appropriate, feasible policy. Instead of analysing the ageing impact, it contrasts previous studies (Babecky and Dybczak, 2009; Braz *et al.*, 2013; Kilponen *et al.*, 2006) by questioning fiscal measures necessary to maintain the current levels of well-being and income.

The conclusion of this study thus answers the following research question: Which fiscal policy measure is the most suitable one for stabilizing the forecasted negative effect of the population ageing on the Czech income and well-being?

The questioned fiscal policy tools follow pension reform recommendations from preceding studies. Both, Babecky and Dybczak (2009) and Kilponen *et al.* (2006) find an increase in the statutory retirement age as insufficient to eliminate the ageing effects on the Czech economy; but differ in the proposed policies. The first article supported by research from other countries (Cristescu, 2009; Gruber and Wise, 2005) suggests a less generous pension system to motivate individuals to work longer and save more money for retirement. Such labour market participation also reduces the poverty risk for retirees (Cristescu, 2009). Peeters and Groot (2015) discuss the benefits of incentivizing the higher participation rate for the ageing European labour supply. Kaliciak *et al.* (2019) elaborate on the Babecky and Dybczak (2009) idea and analyse incentives for demanding voluntary pensions.

In contrast to Babecky and Dybczak (2009), Kilponen *et al.* (2006) recommend a fall of replacement rate. Braz *et al.*'s (2013) findings for Portugal

[1] Data available at https://ec.europa.eu/eurostat.

support positive GDP effects from following such policy of lower replacement rate. In summary, the most discussed ways to mitigate the ageing consequences include postponing retirement, reducing social security payments, and increasing pensions. This study examines the effectiveness of all three policy measures to mitigate the ageing impact. Based on the previous studies, we expect that, as the most advantageous policies, they are designed to boost the labour market participation.

The complexity with which the ageing phenomenon affects economic relations necessitates a research method that can capture economic inter-linkages. The Dynamic Stochastic General Equilibrium (DSGE) model is appropriate for this task. Its high-dimensional structure and ability to model interdependences among variables explain its wide acceptance among authorities worldwide. And there can hardly be a more suitable DSGE model to analyse the Czech fiscal policy than the Aliyev *et al.* (2014) model developed by the Czech Ministry of Finance. The presented modifications will further elaborate on the fiscal-policy aspects of the model for the ageing phenomenon. Such straightforward reformulation aims to motivate the fiscal authorities to use their model also for the demographic agenda.

The applied model originates in our previous study (Gawthorpe and Safr, 2019). In contrast to the previous research, the model also simulates income deterioration subject to ageing and questions the most sustainable fiscal policy. The fiscal-policy measures enter the model in the form of shocks. The shocked variables consist of the social security benefits, the VAT tax, the pensions, the labour supply, and the replacement ratio.

The rest of the paper is as follows. The first section describes the DSGE model construction, followed by the second section, which presents the data for the model calibration. The third section lays down the simulation results, and the fourth section discusses the policy recommendations. The conclusion summarizes the findings at the end of the paper.

1. Model

Dynamic Stochastic General Equilibrium (DSGE) models are utilized by most central authorities around the world and are included among the most common tools for economic forecasting and policy evaluation (Aliyev *et al.*, 2014; Ambriško *et al.*, 2012; Motto *et al.*, 2010; Smets and Wouters, 2007). Several studies have proven their relevance for forecasting and policy assessment. For example, Smets and Wouters (2007) show the ability of a DSGE model to outperform Bayesian Vector Autoregressive (BVAR) models. In addition, numerous studies recommend the general equilibrium framework for studying the ageing phenomenon (Braz *et al.*, 2013; Faruqee and Mühleisen, 2001; Kilponen *et al.*, 2016; Kinnunen, 2008; Ruppert and Stähler, 2022).

The DSGE models reflect the dynamic relationship between economic variables and capture their mutual dependences. The ageing phenomenon simultaneously affects multiple inter-connected economic areas, making these models a perfect fit for modelling the demographic shifts. In contrast, according to Kilponen *et al.* (2016), a model that misses linkages among labour-supply, wages, and consumption undervalues the ageing impact. Similarly, Braz *et al.* (2013) and Babecky and Dybczak (2009) study demographic changes with a complex agent-based model to overcome the potential undervaluation problem. In this study, the computable general equilibrium model is an extended version of the Czech Ministry model (Aliyev *et al.*, 2014) that builds on our previous findings (Gawthorpe and Safr, 2019).

Like Gawthorpe and Safr (2019), the model abstracts from the assumption of the Non-Ricardian agents present in Aliyev *et al.* (2014). The fraction of the rule-of-thumb households in the Euro Area accounts for only 25 percent that is insufficient to obtain a positive consumption reaction to wage growth (Coenen and Straub, 2005). To evade this problem for the model in Aliyev *et al.* (2014), the model presents a different consumption function (see Gawthorpe and Safr, 2019).

The main extension from the Ministry version concerns the model disaggregation procedure to account for four ageing cohorts where the primary breadwinner ranges in groups from 30-39, 40-49, 50-59, or more than 60 years old. In the model terms, these four age groups *r* consist of the young generation *y*, the middle-aged working generation *m*, the older individuals' generation *o*, and the pensioners' generation *p*. The model variables, VAR_t^r, namely consumption, labour supply, labour demand, wages, and overall welfare function, are subject to disaggregation into these four cohorts as follows:

$$VAR_t = (\sum_{r=1}^{A} VAR_t^r \Omega^r) POP \quad (1)$$

where $\sum_{r=1}^{A} \Omega^r = 1$; capital letter A stands for the share of various age groups (*y, o, m, p*), and *POP* expresses the total population, which is assumed to be constant.

This disaggregation concerns all the model blocks and enables evaluating the three fiscal-policy measures as potential welfare and income stabilization tools. Namely, these policies consist of postponing retirement, reducing social-security payments, and increasing pensions.

This extended Ministry model assimilates our previous study (Gawthorpe and Safr, 2019, see the Appendix). The difference stems from the objective of the presented research to study the labour income impact of the demographic shifts and the fiscal-policy assessment. The model also calculates the differentiated income and well-being sensitivity across ageing cohorts and measures the fiscal costs of maintaining the labour income/well-being unchanged in the presence of ageing. The results also compare if the income or the well-being objective is more fiscally feasible.

The model understands the well-being as the weighted aggregate utility $U_t = (\sum_{r=1}^{A} U_t^r \Omega^r)$ with the weights Ω^r reflecting the share of the *r*-th age cohort in the total population. The formulation of well-being as an additive utility function follows the studies from Fleurbaey (2009) and Benjamin *et al.* (2012). The income variable corresponds to the households' earnings $(1-\tau_t^w-\tau_t^{sc})W_t^r N_t^r$ where τ_t^w reflects the income tax, τ_t^{sc} the social security payments, W_t^r the real wage and N_t^r the labor supply for the r-th age cohort.

Analysing the appropriate fiscal tools requires a richer set of shocks to our original version (Gawthorpe and Safr, 2019). The model understands the policy of postponing retirement as the labour supply shock for the elderly, the increasing pensions as the pension shock, and the reduction of the social-security payments as the shock for the social-security variable. The model measures the policies' impact on the government balance with new shocks for the VAT tax, the pensions, and the government transfers.

The applied methodology is as follows:

- First, the model reveals how the ageing phenomenon affects the income and the well-being. The simulation process captures the projected ageing as an exogenously varying share for all ageing cohorts.
- Second, we need to calculate the difference in the values for the well-being/income variables between the scenarios with and without the ageing phenomenon. The model simulations then calculate the necessary size of exogenized fiscal policy variables to eliminate such differences, see Table 1.

Table 1. Matching fiscal policy and model variables

Policy	Variable to implement the shock
Postponing retirement	Increase in the labour supply for the elderly N_t^p
Reducing social-security payments	Shock into the social-security payments τ_t^{sc}
Increasing pensions	Exogenize and increase the pensions PEN_t

Source: Author's representation

Third, this part focuses on the negative effect of the scenario with the lower social-security payments on the government balance. In this final step, the model analyses the size of a different fiscal mix consisting of the VAT tax (τ_t^c), the transfers (TR_t), and the pensions (PEN_t) to avoid this government imbalance on the way that will stabilize the income and the well-being. The applied method is again a conditional forecast.

2. Data

The Czech Statistical Office and the Ministry of Finance in the Czech Republic supply data for this study[2]. Data are available in a quarterly frequency.

[2] Data available at www.czso.cz, www.mfcr.cz.

While the parameters in Table 1 in the Appendix assimilate the benchmark model from Aliyev *et al.* (2014), the model disaggregation requires an extension of the original parameter set.

The parameter $\sigma^{i,r}$ reflects the importance of the income effect relatively to the inter-temporal effect for individual age groups *r*. Linear regression models provide the values by estimating a simple relationship between the consumption, the net income, and the nominal interest rate variables for every age cohort. The net income approximates the income effect, and the interest rate reflects the inter-temporal effect (see the Euler equation 11). Data for the differentiated consumption and the net income originate in the Household budget survey from the Czech Statistical Office and data for the interest rate (3M Pribor) from the Ministry of Finance Database.

The parameters Ω^r represent the share of the individual age cohorts r=y, m, o, p in the total population. The parameter ζ exceeds one, which reflects the substitution character for differently aged labor groups. The parameters ζ^r signal the dependence of the labour groups' demand on their respective wages. The parameter g^r demonstrates the importance of the age differentiated productivity shock and the parameter value approximates the marginal product of labor.

3. Results

This section summarizes the simulation outcome. The study analyses two policy objectives: well-being and labour income.

The Eurostat presents the shares for different age groups for 2018 and the demographic projections for 2050. The values in between these years are assumed to follow a linear function. Table 2 presents the individual shares as utilized for the model simulations.

The income effect of the projected ageing is visible in Figure 2 in the Appendix. The modelling process assimilates a sensitivity analysis where every quarter, we simulate the model by assuming a new share for every age cohort. The curves on individual plots in Graph 2 compare the income dynamics for the scenarios with (grey line) and without (black line) the projected ageing, where the ageing scenario respects the varying shares for the age cohorts (for the parameter values, see Table 2).

Figure 2 suggests a significant reduction of the income for all age groups subject to the demographic changes. This effect originates in the lower productivity of the elderly that translates into lower demand and lower offered wages for this older cohort. Firms respond to the lower labour productivity and subsequent production drops by demanding less labour. Consequently, the lower demand for employees translates into wage deflation for all cohorts. The wage-drop results in the income decrease illustrated in Figure 2.

Table 2. Projections for individual age cohorts

Year	Young	Middle	Old	Retired
2019	0,158	0,145	0,1279	0,247
2020	0,157	0,145	0,1276	0,251
2021	0,156	0,144	0,1272	0,255
2022	0,155	0,144	0,1269	0,260
2023	0,154	0,143	0,1266	0,264
2024	0,153	0,143	0,1263	0,269
2025	0,152	0,142	0,1259	0,273
2026	0,151	0,142	0,1256	0,277
2027	0,149	0,141	0,1253	0,282
2028	0,148	0,141	0,1249	0,286
2029	0,147	0,140	0,1246	0,291
2030	0,146	0,140	0,1243	0,295
2031	0,145	0,139	0,1239	0,300
2032	0,144	0,139	0,1236	0,304
2033	0,143	0,138	0,1233	0,308
2034	0,142	0,138	0,1230	0,313
2035	0,141	0,137	0,1226	0,317
2036	0,140	0,137	0,1223	0,322
2037	0,139	0,136	0,1220	0,326

Source: Author's calculations

Figure 3 also finds the projected demographic shifts for the Czech economy to exacerbate the Czech well-being. Figure 3 documents such a pattern. The estimation procedure for Graph 3 stays like the one from the previous chart. Similar to Graph 2, this outcome stems from iterating the model for every quarter with the new values for the shares of individual age groups. The close relationship between consumption and well-being explains the findings. The income reduction resulting from the ageing phenomenon (Figure 2) constrains households' consumption and results in lower well-being.

From the selected age cohorts, retirees seem to be the most sensitive group to ageing. In contrast to other groups, the utility drop for pensioners would be two-fold. First, the higher share of less-productive pensioners would negatively affect aggregate wage. This effect would limit consumption, and in turn, well-being. Second, the increasing number of retirees would reduce pension size per retiree, adversely impacting their consumption and well-being. Similarly, the study from El Salvador (Peña, 2020) discusses declining consumption for the ageing population, and the study (Cristescu, 2009) questions poverty risk after retirement with panel data for the EU countries. This risk seems to be sensitive to the labour participation of the elderly.

The aggregate well-being and income effects are visible in Figure 4 in the Appendix. Overall, the income seems to decline faster from its benchmark in comparison to the well-being variable. This differing effect is an essential finding for policy makers as it would require more extensive measures to stabilize labour income rather than well-being. In summary, well-being is a more feasible objective to target. The next part of the study analyses different fiscal policies to maintain income and well-being resistant to the ageing phenomenon.

3.1. The main linkages in the model

The DSGE model's complexity motivates this sub-section revealing the flow of the shock impact throughout the model. This simplified illustration in Figure 5 in the Appendix clarifies the main inter-linkages, although it insinuates causal relationships while the model operates with endogenous relations.

The first line of the diagram reflects the labour demand for the retirees. Firms demand this low-productive cohort less than the other age groups, which translates into lower wages for the retirees (see equation 19).

The second line signals a chained effect from increasing the elderly share in the model. The more the pensioners' share increases, the more their wage impacts the aggregate wage level (see equation 15). As apparent from the first line, the lower wage for this cohort reduces the aggregate wage and pushes up the relative wages for the remaining groups. Firms consider the growing relative wages by decreasing their labour demand (see equation 23). The drop-in demand for employees and overall lower earnings adversely affects households' spending decisions, and in turn, their well-being shrinks. This income and well-being reaction correspond with Figure 2 and 3.

The third to the fifth lines of the diagram illustrate the three studied fiscal policy measures targeting constant well-being and income. The fiscal-policy tools consist of the postponement of retirement, the reduction of social security payments, and the increase of pensions.

The model simulates the postponement of retirement as the growth of labour supplied by the retirees' cohort. Despite the lower productivity of older generations, their labour supply growth positively affects their disposable income. The higher earnings motivate this elderly group to increase their well-being by consuming more.

Another fiscal compensation mechanism would incentivize individuals to supply more labour by reducing their mandatory social-security payments. Such policy increases individuals' net wages and reduces firms' labour costs. Subsequent employment growth results in higher net income for all individuals that can now improve their well-being by buying relatively more goods and services.

The last analysed scenario suggests facing the growing elderly share by providing pensions for more individuals. This policy would directly impact the consumption and subsequently the well-being of pensioners.

3.2. Postponing retirement or increasing pensions?

Table 3 presents the necessary size of the labour supply/pensions to maintain the well-being and the income level indifferent to ageing. The Utility difference and the Income difference columns contain values for the demographic effect on the aggregate well-being and income levels in percentage points.

Table 3. Ways to maintain the same level of well-being or income

Years	Constant Well-being			Constant Income		
	Utility diff.	Labour supply	Pension	Income diff.	Labour supply	Pension
2020	0,00	0,71	0,15	-0,01	0,14	0,53
2021	-0,02	2,41	0,38	-0,06	0,37	1,27
2022	-0,07	5,51	1,87	-0,24	1,16	2,00
2023	-0,10	8,07	2,92	-0,46	2,06	2,89
2024	-0,11	9,91	3,56	-0,49	1,96	4,08
2025	-0,17	16,68	5,04	-0,70	2,91	5,85
2026	-0,20	22,22	5,54	-0,84	3,41	8,01
2027	-0,25	30,37	6,20	-0,96	3,73	10,51
2028	-0,22	32,74	5,30	-1,02	3,93	11,80
2029	-0,18	34,53	4,08	-0,65	2,39	12,75

Note: Every value represents a quarterly average over a year. All values are in percentage points.
Source: Author's calculations

Labour supply columns reflect the situation of postponing the retirement age. The model simulates this scenario as an exogenous shock into the retirees' labour supply, necessary to maintain either the well-being or the income unaffected by ageing. In other words, it is a conditional forecast for the pensioners' labour supply conditional on the unaltered well-being/income dynamics. The forecasted well-being/income values are differences between the scenario with and without the projected demographic shifts, see Table 3.

Increasing the labour supply variable for the retirees corresponds with prolonging their working age. However, their productivity would be lower compared to younger generations. Individuals would have to compensate for the smaller productivity by working more hours, which explains the sizable labour supply shift necessary to maintain the constant well-being in Table 3. For comparison, the relatively smaller labour supply growth stabilizes the income and stems from the direct link between the number of hours worked and individuals' income.

Another fiscal-policy option would be to increase pensions. The simulation process is a conditional forecast for the aggregate pension variable. In other words, this forecast measures the necessary pension growth to stabilize the targeted variables. In contrast to the previous measure of postponing the retirement age, pensions would need to grow more to stabilize the income rather than the well-being. Pensions affect the elderly's consumption directly and their income indirectly; the close consumption-well-being relationship then explains why the fiscal policy would be less costly if it is targeting well-being.

Overall, these findings prove each of the simulated tools appropriate only for meeting one of the two objectives. Labour supply would have to increase relatively more if targeting the constant well-being and not the income. Oppositely, the fiscal authority would have to provide more pensions if targeting the income rather than the well-being. In other words, this result suggests the same size of the labour supply which is sufficient to maintain constant well-being as insufficient to keep income unaltered by the ageing phenomenon. Similarly, pensions' growth that would meet the well-being objective would not help the income deterioration.

In conclusion, fiscal policy makers should legislate these fiscal measures only if they aim at meeting one of the herein analysed objectives. Fortunately, there exists a third path that could simultaneously stabilize both the projected income and the well-being.

3.3. Social security benefits

This scenario questions the size of reducing social-security payments to stabilize the well-being and the income deterioration. This fiscal policy incentivizes individuals to supply more labour during their productive years. Individuals would gain more income over their lifetime, and the government would benefit from higher collected tax revenue. This tax revenue would partially offset the budget deficit from lower social-security payments.

Table 4 summarizes the required reduction of the social-security payments to revert the deteriorating income trend. The model simulates this scenario as a negative shock for the social-security payments conditional on the income stability. The applied values for the projected income decline are visible in Figure 2.

The first column of Table 4, Income diff., represents the income difference for the scenario with and without the ageing phenomenon. The second column illustrates the social security decline as a feasible option to preserve the total income indifferent to the demographic shifts; however, at the cost of a significant government balance deficit visible in the third column.

The last three columns summarize findings for selected fiscal-policy measures as ways to stabilize the government's budget. First, the fiscal authority could balance the budget by reducing pensions for the elderly. The pensions' drop would have to be sizable as pensions' account for less than a quarter of the Czech government expenditures, plus the pensions' reduction negatively affects net income for the elderly and subsequently the government revenue. Nevertheless, the pension drop would incentivize individuals to work harder and become more responsible for their future income.

Table 4. Decrease of social security payments to keep income constant in p. b.

Years	Income diff.	Social sec.	Gov. balance	Pension	VAT, PEN, TR	VAT
2020	-0,01	-0,04	-0,02	-0,22	0,00	0,14
2021	-0,06	-0,20	-0,19	-1,50	-0,17	0,64
2022	-0,24	-0,51	-0,44	-3,83	-0,56	1,54
2023	-0,46	-0,76	-0,57	-5,82	-0,85	1,97
2024	-0,49	-0,80	-0,54	-7,00	-0,90	1,54
2025	-0,70	-1,27	-0,92	-10,95	-1,55	2,63
2026	-0,84	-1,55	-1,24	-16,16	-2,20	3,93
2027	-0,96	-1,82	-1,44	-21,52	-2,61	4,81
2028	-1,02	-1,92	-1,72	-28,94	-3,26	6,01
2029	-0,65	-1,20	-1,02	-31,05	-2,45	3,66

Note: Values in individual columns are quarterly averages for every year. All values are in percentage points.
Source: Author's calculations

Second, higher VAT taxation could help reduce the government deficit. The advantage of this indirect taxation is its non-distortionary character. The VAT column displays the taxation values necessary to stabilize the government budget for this unaltered income scenario. Third, the fifth column provides absolute values for a policy mix consisting of the same level of increase for the VAT taxation and level of decrease for the pensions and other transfers.

Table 5. Decrease of social security payments to keep constant well-being

Years	Well-being	Soc security	Gov. balance	Pension	VAT, PEN, TR	VAT
2020	0,00	-0,17	-0,10	-0,74	0,23	0,43
2021	-0,02	-0,34	-0,26	-1,77	0,53	0,90
2022	-0,07	-1,02	-0,67	-3,94	-1,34	2,39
2023	-0,10	-1,47	-1,07	-7,16	-1,57	3,80
2024	-0,11	-1,54	-0,90	-7,44	-2,04	2,91
2025	-0,17	-2,42	-1,54	-12,69	-2,97	4,91
2026	-0,20	-2,75	-1,78	-17,83	-3,33	5,96
2027	-0,25	-3,40	-1,97	-23,41	-4,19	6,81
2028	-0,22	-2,92	-2,12	-31,11	-3,10	7,61
2029	-0,18	-2,42	-1,29	-32,49	-3,13	4,68

Note: Values in individual columns are quarterly averages for every year. All values are in percentage points.
Source: Author's calculations

Table 5 in the Appendix presents results for fiscal policies targeting constant well-being. The values assimilate the situation for targeting the stable income in Table 4. The first column, Well-being, illustrates the well-being drop as the consequence of the ageing phenomenon. The social security measure necessary to stabilize the well-being seems like a viable option as shown in the third column.

The values for the social-security payments' drop appear a bit more sizable to stabilize the well-being rather than the income deterioration. This small divergence originates in the direct effect of the social-security payments on net income.

In summary, this fiscal measure appears the most fiscally sustainable from the analysed policies. This policy tool would have to be the least sizable to stabilize the income and well-being.

4. Discussion

The model simulations show a significant impact of the ageing phenomenon on the aggregate income and the total well-being in the Czech economy. This outcome supports a negative effect of ageing published in previous studies (Babecky and Dybczak, 2009; Braz *et al.*, 2013, p. 20; Faruqee and Mühleisen, 2001; Gawthorpe and Safr, 2019). In line with the study of Kilponen *et al.* (2016), the DSGE model proves to be capable of capturing the interdependences among multiple variables affected by the demographic changes, see Figure 5.

This study offers an analysis of the three fiscal-policy measures to maintain the original level of income and well-being indifferent to the ageing phenomenon. The first two fiscal measures prove sustainable only at meeting one of these objectives. First, the postponement of retirement seems to be a feasible way to keep the labour income unaltered but not to stabilize the well-being. In comparison, Babecky and Dybczak (2009) find the increase in the statutory retirement age insufficient to eliminate the impact of ageing on the Czech economy. Kilponen *et al.* (2006) conclude for Finland that increasing the effective retirement age is financially unsustainable. Second, the increase in pensions could help stabilize the well-being but would have to be too large to stabilize the income level.

Finally, the social security measure appears to be the most beneficial policy tool for meeting objectives, the constant well-being, and income. This policy tool incentivizes people to work more during their younger years. Such a close relationship between the social security payments and labour market participation supports the study from Gruber and Wise (2005). The subsequent increase in the labour supply would lessen the ageing effects. Kinnunen (2008) also recommends the stimulation of employment to alleviate the ageing impact in Finland.

Following preceding research, the inevitable pension reform and the consequences of ageing negatively affect fiscal sustainability (Faruqee and Mühleisen, 2001; Langenus, 2006; Oshio and Oguro, 2013; Peeters and Groot, 2015;

Peña, 2020; Ramos-Herrera and Sosvilla-Rivero, 2020). The previous section complements these studies and reveals the government's deficit resulting from the social-security change necessary to stabilize income and well-being. The findings recommend balancing the government budget with a fiscal-policy mix consisting of the VAT tax rate increase and the decrease of pensions and other transfers. Faruqee and Mühleisen (2001) conclude with similar findings for Japan. The authors recommend stabilizing the government budget in the presence of ageing by increasing consumption tax and reducing social security benefits.

Overall, the reduction of the social security payments financed by the suggested policy mix, which includes the drop of pensions, would increase individuals' responsibility for their future income while motivating them towards higher productivity during their younger years. These findings support the study of Babecky and Dybczak (2009) that also discusses the lower impact of the ageing phenomenon on the Czech economy when the public pension system is less generous.

Conclusions

The severity of the projected ageing for the Czech economy motivates this study to search for fiscally sustainable policy measures to stabilize the expected drop in the overall income and well-being. The applied method is an extended Dynamic Stochastic General Equilibrium (DSGE) model initially developed by the Czech Ministry of Finance. The model elaboration helps fiscal policy makers to utilize the model also for impact-evaluation of the demographic agenda.

This study suggests disaggregation of the original model to account for four differently aged groups, the young generation, the middle-aged working generation, the older individuals, and the pensioners. The new model version also assumes varying labour productivity across the selected age cohorts that translate into varying labour income, where pensioners are assumed to receive labour income and pensions, as well.

The simulation process in the first step applies the projected shares for the age groups to calculate the ageing effects on the income and the well-being variables. In contrast to the previous research, the presented study does not stop here as it continues to search for the required boost of these two variables to prevent them from deteriorating. In this second step, we iterate the model to find the size of individual fiscal policy measures conditional on returning these declined income and well-being variables to their original size.

The model findings prove different ageing effects on the labour income than on the well-being. The income would deteriorate more than the individuals' well-being in the presence of the projected ageing. This difference could incentivize policy makers to select one of these objectives and use appropriate fiscal measures. The three herein analysed policy tools would target the objectives as follows. First, the postponement of retirement would shorten the number of years for which individuals receive their pensions. Second, the decrease of social-security payments would incentivize individuals to work more during their productive years. Third, the pensions' increase would provide access for the growing share of retirees to this non-labour income.

The results suggest reducing social-security payments as the most fiscally sustainable tool to meet both objectives simultaneously. The postponement of retirement appears more advantageous for the policy targeting constant income and the increase of pensions for targeting well-being. However, the scenario with lower social-security benefits would lead to a growing government deficit. This study simulates three ways to balance the government budget by increasing the VAT tax, reducing pensions, or a policy mix. The results favour the fiscal-policy mix, which combines the increase of non-distortionary taxation such as VAT, the reduction of pensions, and other transfers. The next research could question the political feasibility of the suggested fiscal policy. It could also analyse other fiscal-policy measures to reduce the ageing effects on both the well-being and the income.

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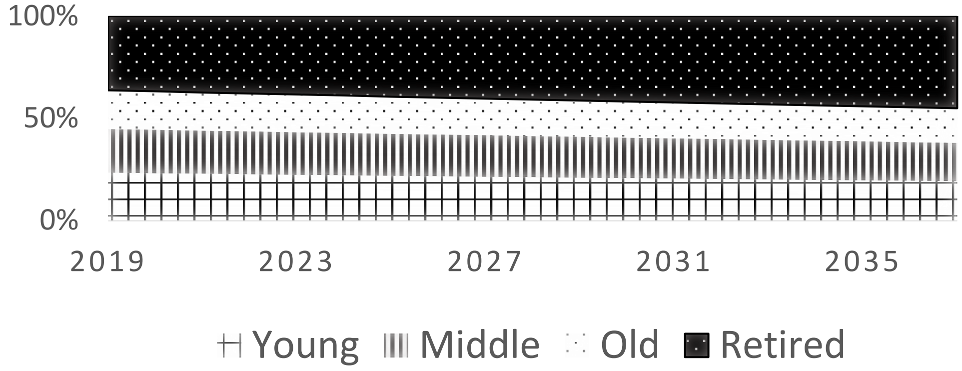
Appendix

Figure 1. Evolution of shares for individual age cohorts



Note: Dataset is available on the Eurostat (2019) website. Young group ages 30-39 years old, middle-aged 40-49 years old, old ones are 50-59 years old and retired are older than sixty.
Source: Author's representation

Table 1. Parameter values

μ_{im}	μ_{cm}	θ^{c}	θ^{c*}	θ^{i}	θ^{i*}	ξ^{w}	ξ^{p}	β	λ_{π}	λ_{y}	η
0.32	0.22	0.3	0.3	0.2	0.2	0.2	0.9	0.99	1.5	0.5	0.53

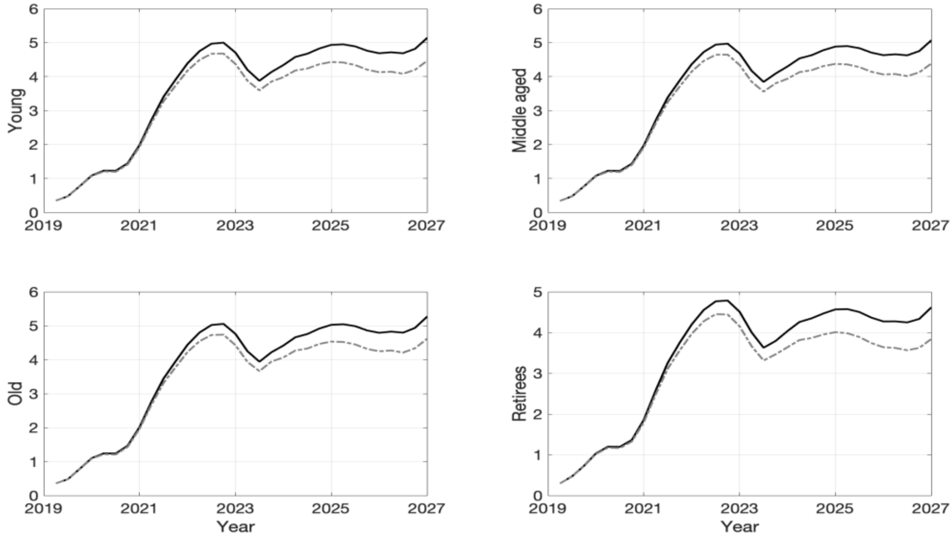
θ^{w}	κ	ρ	ψ^{N}	$\emptyset_{r}$	δ	h	ω_{ggc}	ω_{w}	ω_{c8}	α_{R}	s_{r}
2	11	-log(β)	1.55	0.52	0.1	0.8	0.25	0	0.594	0.465	1.531

σ^{i}	ξ^{pm}	ξ^{px}	Ω^{y}	Ω^{o}	Ω^{m}	Ω^{p}	$\sigma^{i,y}$	$\sigma^{i,m}$	$\sigma^{i,o}$	$\sigma^{i,p}$	ζ^{y}
8.665	0.678	0.673	0.158	0.145	0.128	0.247	5.338	4.748	5.176	4.735	2.138

ζ^{m}	ζ^{o}	ζ^{p}	ζ	g^{y}	g^{m}	g^{o}	g^{p}
2.008	2.386	1.285	2	1.069	1.004	1.193	0.643

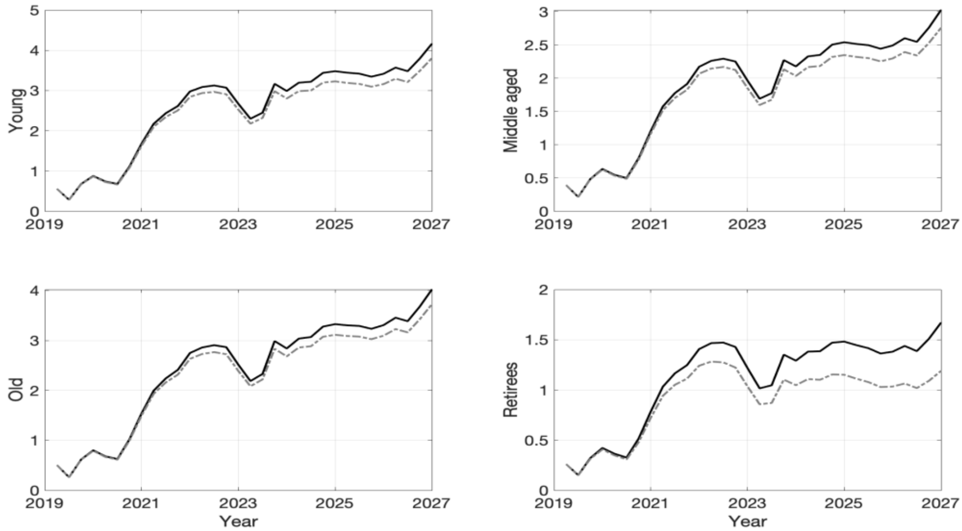
Source: Author's calculations

Figure 2. Heterogenous income



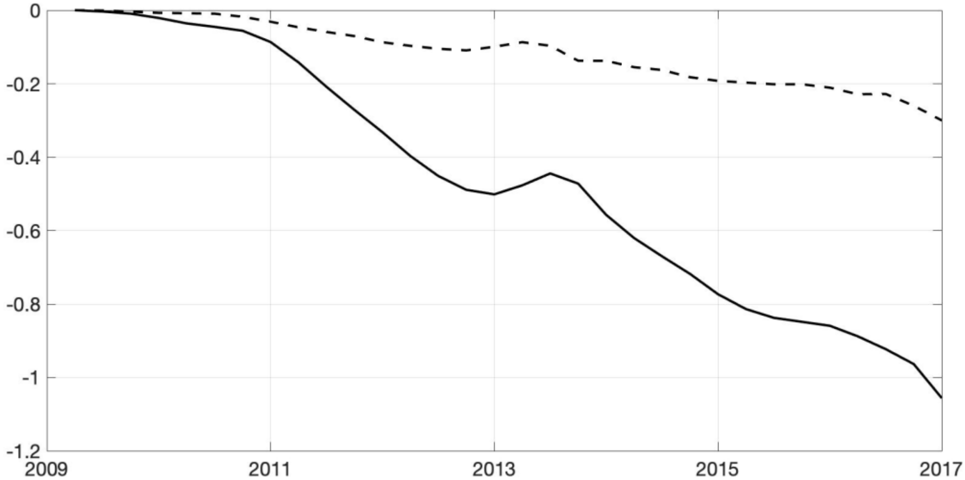
Note: The values are in percentage points. The y-axis presents values in percentage points. The black line reflects the scenario without the ageing phenomenon and the grey dashed line the situation subject to the demographic shifts.
Source: Author's representation

Figure 3. Heterogenous well-being



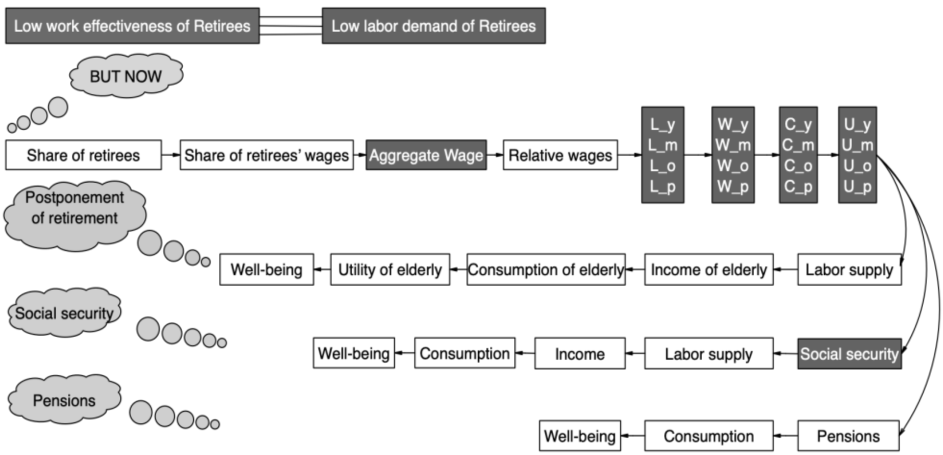
Note: The values are in percentage points. The y-axis presents values in percentage points. The black line reflects the scenario without the ageing phenomenon and the grey dashed line the situation subject to the demographic shifts.
Source: Author's representation

Figure 4. Utility and Income decline due to ageing



Note: The values are in percentage points. The y-axis presents values in percentage points. The dashed line stands for the decreasing utility over time with ageing as compared to the situation without the demographic shifts. The full line depicts a comparison between income in presence of ageing and income in the scenario without ageing.
Source: Author’s representation

Figure 5. Interlinkages in the model



Note: The grey fill colour in a box means a reduction of a variable.
Source: Author’s representation

Methodological note on the estimated DSGE model

1. MODEL

1.1. Households

Households are divided into four differently aged groups *r*: the young generation *y*, the middle-aged working generation *m*, the older individuals *o*, and the pensioners p. Variables for these age groups VAR_t^r, namely consumption, labor supply, labor demand, wages, and overall welfare function, can be aggregated as follows:

$$VAR_t = (\sum_{r=1}^{A} VAR_t^r \Omega^r)POP \tag{1}$$

where $\sum_{r=1}^{A} \Omega^r = 1$; capital letter A stands for the share of various age groups (*y*, *o*, *m*, *p*), and *POP* expresses the total population, which is assumed to be constant.

A household of type $r\epsilon(y,o,m,p)$ maximizes its expected infinite horizon utility over consumption $C_{j,t}^r$ and labor supply $N_{j,t}^r$:

$$E_t \sum_{n=0}^{\infty} \beta^n U_{j,t+n} = E_t \sum_{n=0}^{\infty} \beta^n \left\{ log\left(C_{j,t+n}^r - H_{j,t+n}^r\right) - \frac{\left(N_{j,t+n}^r\right)^{1+\psi_N}}{1+\psi_N} \right\} \tag{2}$$

where $H_{j,t+n}^r = h_r C_{j,t+n-1}^r$ stands for the external habit formation, β labels the discount factor, and the parameter ψ_N is the inverse elasticity of labor supply; subject to the intertemporal budget constraint:

$$(1+\tau_t^c)P_t^c C_{j,t}^r + P_t^I I_{j,t} + P_t a\left(u_{j,t}\right)K_{j,t}^s + \frac{1}{R_t}B_{j,t+1} + \frac{1}{R_t^*+rp_t}S_t B_{j,t+1}^* = B_{j,t} + S_t B_{j,t}^* + \left(1-\tau_t^k\right)\left[R_t^k u_{j,t} K_{j,t}^s + Q_t\right] + [1-\tau_t^w - \tau_t^{sc}]W_t^r(\Lambda_t^r)^{g^r} N_{j,t}^r + \tau_t^b W_t^b (N_{j,t} - L_{j,t}) + TR_t^r + PEN_t^r. \tag{3}$$

The expenditure side of the constraint includes the consumption expenditure $P_t^c C_{j,t}^r$ increased for the VAT tax τ_t^c, the investment variable $I_{j,t}$ multiplied by the price of the investment P_t^I, the capital stock expenditures $P_t a\left(u_{j,t}\right)K_{j,t}^s$ with the capital adjustment costs $a\left(u_{j,t}\right)$ and purchases of the domestic $B_{j,t+1}$ and the foreign bonds $B_{j,t+1}^*$ that are denominated in the Czech currency with the exchange rate S_t. Variable R_t labels the nominal interest rate, R_t^* stands for the foreign interest rate, and finally rp_t is the risk premium for holding foreign bonds.

The income side of the constraint consists of the income from holding domestic and foreign bonds, the capital income from earning the rental rate R_t^k on capital $K_{j,t}^s$ with the utilization rate $u_{j,t}$ and the portion of profit Q_t from owning a firm. These capital gains are deduced for the capital tax τ_t^k. A representative household also earns the labor income $W_t^r N_{j,t}^r$ increased for the age group r productivity Λ_t^r and the productivity shock g^r across age cohorts (see Babecký & Dybczak, 2009). The labor income is subject to the income tax τ_t^w as well as the payments for the social security benefits τ_t^{sc}. The low-income class also obtains unemployment benefits τ_t^b dependent on the aggregate wage bill W_t^b and unemployment rate $(N_{j,t} - L_{j,t})$. The other transfers TR_t^r include the government transfers other than the unemployment benefits τ_t^b and the pensions PEN_t^r.

Every household with a retired member becomes eligible for a pension. The pensions are a function of the net wage $W_t^{net,r}$, the labor supply N_t^r for a particular age group r, and the replacement rate rep_t:

$$PEN_t^r = rep_t W_t^{net,r} N_t^r. \tag{4}$$

The model assumes the government's full control over the replacement rate and that the net wage equals to:

$$W_t^{net,r} = [1 - \tau_t^w - \tau_t^{sc}] W_t^r. \tag{5}$$

The utility maximization problem above results in similar first-order conditions to the benchmark model (Aliyev *et al.*, 2014) except for the consumption function that assimilates the study from Gawthorpe and Safr (2019). The derivation procedure for the consumption function results in a direct relationship between labor income and consumption variable. While this derivation method keeps the same representation of the utility function (2) and the budget constraint (3), it assumes representative agents to face different steps when making consumption decisions.

The agent selects from a bundle of expenditures and purchases it with finances acquired by liquidating assets from her income bundle. When the agent chooses the consumption as a preferable expense, she compares the price and the marginal utility from consuming a good. The nuanced method requires redefining the budget constraint as a bundle of different expenditures $\mathbb{Z}_t$ and income sources $\mathbb{R}_t$, before deriving the FOC for consumption:

$$\mathbb{Z}_t^r = \mathbb{R}_t^r \tag{6}$$

The individual expenditure variables l sum up to the original expenditure bundle:

$$\mathbb{Z}_t^r = \sum_{l=1}^{n} (\mathbb{Z}_t^r)^l. \tag{7}$$

where $l = c, i, k, b, b^*$ represent the variables in the expenditure side of the budget constraint. For example:

$$\mathbb{Z}_t^c = \begin{pmatrix} (1+\tau_0^c)P_0^c C_{j,0} \\ (1+\tau_1^c)P_1^c C_{j,1} \\ . \\ . \\ . \\ (1+\tau_T^c)P_T^c C_{j,T} \end{pmatrix}, \mathbb{Z}_t^i = \begin{pmatrix} P_0^I I_{j,0} \\ P_1^I I_{j,1} \\ . \\ . \\ . \\ P_T^I I_{j,T} \end{pmatrix}, \text{where } t = 1,2,\dots T \tag{8}$$

and similarly, we can rewrite the expenditures for capital stock, domestic and foreign bonds. The income bundle consists of different sources of income:

$$\mathbb{R}_t^r = \sum_{\varpi=1}^n (\mathbb{R}_t^r)^{\varpi}. \tag{9}$$

where $\varpi = b, b^*, k, q, w$ are the variables in the income side of the budget constraint.

The above simplified reformulation of the budget constraint necessitates one more step to represent a constant elasticity of substitution (CES) function. Each side of the equation (6) is multiplied by the fraction $\frac{1}{1-\sigma_i^r}$ and set to the power of $1-\sigma_i^r$:

$$\frac{1}{1-\sigma_i^r}\left(\sum_{l=1}^n (\mathbb{Z}_t^r)^l\right)^{1-\sigma_i^r} = \frac{1}{1-\sigma_i^r}\left(\sum_{\varpi=1}^n (\mathbb{R}_t^r)^{\varpi}\right)^{1-\sigma_i^r}. \tag{10}$$

This transformation reflects a household's option to choose from various types of expenditure and different sources of income. The household views different forms of income or expenses as complementary or substitutes to some degree.

Finally, the maximization of the utility with respect to the generalized budget constraint (10) results in the following consumption function (see Gawthorpe and Safr, 2019):

$$\frac{1}{C_t^r - hC_{t-1}^r \frac{1}{uz_t}} - \beta h E_t \frac{1}{C_{t+1}^r uz_{t+1} - hC_t^r} = \lambda_t P_t^c (1+\tau_t^c)(\mathbb{R}_t^r)^{-\sigma_i^r} \tag{11}$$

where $\lambda_t = \frac{\lambda_t^z}{P_t}$

and

$$\mathbb{R}_t^r = B_t + S_t B_t^* + \left(1-\tau_t^k\right)\left[R_t^k u_t K_t^s + Q_t\right] + (1-\tau_t^w - \tau_t^{sc}) W_t^r \Lambda_t^r N_{j,t}^r + \tau_t^b W_t^{b,r}(N_{j,t}^r - L_{j,t}^r) + TR_t^r + PEN_t^r. \tag{12}$$

Log-linearization of the income equation results in:

$$\widetilde{\mathbb{R}}_t^r = \omega^{b,r}\tilde{B}_t + \omega^{b,r*}\left(\tilde{S}_t + \tilde{B}_t^*\right) + \omega^{t_k,r}\tau_t^k + \omega^{k_s,r}\left[\tilde{R}_t^k + \tilde{u}_t + \tilde{K}_t^s\right] + \omega^{q,r}\tilde{Q}_t + \omega^{tn,r}\tau_t^w + \omega^{tsc,r}\tau_t^{sc} + \omega^{n,r}\left(\widetilde{W}_t^r + g^r\widetilde{\Lambda}_t^r + \widetilde{N}_t^r\right) + \omega^{ntb,r}(\tilde{\tau}_t^{b,A} + \widetilde{W}_t^{b,r}) + \omega^{n,r}\widetilde{N}_t^r + \omega^{lb,r}\tilde{L}_t^r + \omega^{tr,r}\widetilde{TR}_t^r + \omega^{pen,r}\widetilde{PEN}_t^r \tag{13}$$

where individual omega parameters express the importance of an income variable for a respective age group *r* in the total income. The derivation of the utility function with respect to consumption assimilates the study of Adolfson *et al.* (2007). The equation (11) combines the significance of the income and the intertemporal effect on consumption, which assimilates the study from Cambell and Mankiw (1989).

The heterogeneous labour supply equation also reflects a first-order condition from maximizing the utility function (2) subject to the budget constraint (3):

$$(N_t^r)^{\psi_N} = \lambda_t[1 - \tau_t^w - \tau_t^{sc}]W_t^r(\Lambda_t^r)^{\not{g}^r}. \qquad (14)$$

The remaining FOCs stay identical to the original study from the Aliyev *et al.* (2014).

1.2. Labour market

The model assumes an imperfectly competitive labour market with sticky wages. The modelling procedure for the wage stickiness assimilates the price stickiness (Erceg *et al.*,2000) from the benchmark model. The only difference concerns the disaggregated utility function (2) and the budget constraint (3), accounting for different age groups *r*.

The wage-staggered environment determines a fraction of households $1 - \xi_w$ select an optimal wage w_t^* for time *t* and the remaining households fix their wage size from the previous period:

$$(W_t^r)^{1-\theta_w} = \xi_w(W_{t-1}^r)^{1-\theta_w} + (1 - \xi_w)(W_t^{r*})^{1-\theta_w}. \qquad (15)$$

Every household j supplies differentiated labor $N_{j,t}$ while a labor bundler anticipates the labor demand from firms L_t. The labor demand is a CES function for differentiated labor supply:

$$E_t(L_t^r) = \left[\int_0^1 \left(N_{j,t}^r\right)^{\frac{\theta_w - 1}{\theta_w}} dj\right]^{\frac{\theta_w}{\theta_w - 1}} \qquad (16)$$

where the elasticity of substitution θ_w parameter for different types of labor must be bigger than one $\theta_w > 1$.

Maximization of the utility function (2) rewritten for time t+k for the household that sets its wage in period *t* with respect to the intertemporal budget constraint (3), constraints (15, 16) and the demand for household's labor:

$$N_{j,t}^{r} = L_{t}^{r}\left(\frac{W_{t}^{r}}{W_{j,t}^{r}}\right)^{\theta_{w}} \tag{17}$$

results in

$$W_{t}^{r*} = \frac{\theta_{w}}{\theta_{w}-1}\frac{\sum_{n=0}^{\infty}(\beta\xi_{w})^{n}(N_{t+n}^{r}(j))^{1+\psi_{N}}}{\sum_{n=0}^{\infty}(\beta\xi_{w})^{n}(1-\tau_{t+n}^{w})\lambda_{t+n}N_{t+n}^{r}(j)}. \tag{18}$$

Log-linearization of the equation (18) under the assumption of (15) and (16) enables computation of the final New Keynesian Phillips curve for wages in a log-linearized form:

$$\widetilde{W}_{t}^{r} = \omega_{wwf}\widetilde{W}_{t+1}^{r} + \omega_{wwl}\widetilde{W}_{t-1}^{r} + \frac{\psi_{N}(1-\xi_{w})(1-\beta\xi_{w})}{p_{ww}}\tilde{L}_{t}^{r} + \frac{(1-\xi_{w})(1-\beta\xi_{w})}{p_{ww}}\left\{\frac{\tau^{w}}{1-\tau^{w}}\tau_{t}^{w} - \lambda_{t}^{z} + P_{t}\right\}. \tag{19}$$

1.3. Firm behaviour

A monopolistic competition, defined by price stickiness, dominates the product market. The model extends the original firm's choice and assumes higher labour demand for more productive employees, where their productivity level depends on the employees' age profile Λ_{t}^{r}. The firm minimizes its total costs

$$\min\sum_{r=1}^{A} W_{t}^{r}L_{t}^{r}\,(1+\tau_{t}^{sc}) + R_{t}^{k}K_{t} \tag{20}$$

with capital K_{t}, social security benefits paid by the firm for its employees τ_{t}^{sc}, and differentiated labor L_{t}^{r} into age cohorts r, subject to the production function

$$Y_{t} = K_{t}^{\alpha}\left(z_{t}\sum_{r=1}^{A}\left((\Lambda_{t}^{r})^{g^{r}}L_{t}^{r\,\frac{\zeta^{r}-1}{\zeta^{r}}}\right)^{\frac{\zeta^{r}}{\zeta^{r}-1}}\right)^{1-\alpha} \tag{21}$$

where

$$L_{t} = \sum_{r=1}^{A}\left((\Lambda_{t}^{r})^{g^{r}}L_{j,t}^{r\,\frac{\zeta^{r}-1}{\zeta^{r}}}\right)^{\frac{\zeta^{r}}{\zeta^{r}-1}}, \tag{22}$$

labour takes the form of a constant elasticity of substitution aggregator, in order to derive the optimum demand for a labour of type *r*:

$$L_{t}^{r*} = \left(\frac{W_{t}}{W_{t}^{r}}\frac{1+\tau_{t}^{sc}}{1+\tau_{t}^{sc,r}}\right)^{\zeta^{r}}\left(\frac{(\Lambda_{t}^{r})^{g^{r}}}{\Lambda_{t}{}^{g}}\right)^{\zeta^{r}-1}L_{t}. \tag{23}$$

The derivation procedure assimilates the Dixit-Stiglitz index for consumption from Galí (2015). A representative firm demands labour from the age group *r* that is relatively less costly in wages and social security payments and is more productive.

1.4. Government

Fiscal-policy makers consider the demographic structure of population when providing pensions:

$$PEN_t^r = rep_t^r[1 - \tau_t^w - \tau_t^{sc}]W_t^r N_t^r. \tag{24}$$

The pensions represent a fraction of the government expenditures GE_t

$$GE_t = PEN_t + TR_t + G_t^c + \tau_t^b(N_t - L_t) \tag{25}$$

that also consist of the transfers TR_t, the government consumption variable G_t^c and the unemployment benefits $\tau_t^b(N_t - L_t)$. The government revenue GR_t finances these expenditures

$$GR_t = \tau_t^c(P_t^c C_t + G_t^c) + \tau_t^w W_t L_t + \tau_t^{sc} W_t L_t + \tau_t^k\left(R_t^k K_t + Q_t\right). \tag{26}$$

The government revenue consists of the VAT tax revenue from private $P_t^c C_t$ and public consumption G_t^c, the labor-income tax revenue τ_t^w, the social security payments τ_t^{sc} and the capital tax revenue τ_t^k. All the taxes are assumed to follow an exogenous AR (1) process.