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Testing the weak form efficiency of the French ETF market with the LSTAR-ANLSTGARCH approach using a semiparametric estimation

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Abstract

The present research aims to test the weak-form efficiency of the French ETF market through a LSTAR model with ANSTGARCH errors, by using semiparametric maximum likelihood where the innovation distribution is replaced by a nonparametric estimate based on the kernel density function. In this paper, we consider the daily Xtrackers CAC 40 UCITS from 2009 to 2020 for the analysis as it is supposed to capture more information compared to other French stock markets. After application of different statistical tests, we show that the price fluctuations appear as the result of transitory shocks and the predictions provided by the LSTAR-ANLSTGARCH model are better than those of other models for some time horizons. The predictions from this model are also better than those of the random walk model; accordingly, the XCAC 40 price is a not weak form of an efficient market for the entire period because its successive return is nonlinearly dependent and does not generate randomly.

Keywords: LSTAR-ANLSTGARCH model, semiparametric maximum likelihood, nonlinearity, market efficiency, kernel density

Introduction

Financial market efficiency is certainly one of the most discussed theories in the financial field. The financial market efficiency hypothesis states that the current prices reflect all available information about the actual value of the underlying assets. However, following the different past crises, there has been a disconnection between the stock price and its fundamental value (Colmant *et al.*, 2009; Huber *et al.*, 2008; Lardic and Mignon, 2006). The asset prices do not reflect the best estimate of agents

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in the market. The idea is based on the importance of predicting future prices and their ability to reflect immediately all available relevant information. In other words, the future stock returns have some predictive relationships with the available information of present and historical stock returns. In this case, the nonlinear models are known to be efficient for financial time series forecasting (Antwi *et al.*, 2019; Franses *et al.*, 2000; Kyrtsou and Terraza, 2003; Ouyang *et al.*, 2020).

The interest in nonlinear time series models has been increasing. The presence of nonlinearity in stock price series has important implications for informational efficiency. Indeed, if a series exhibits a nonlinear structure, this implies significant nonlinear dependencies between the observations (Chikhi and Bendob, 2018). In applications to financial time series, the models which allow for regime-switching behaviour have been most popular, especially the class of smooth transition autoregressive (LSTAR) models, introduced by Teräsvirta (1994). A lot of work in this area has been devoted to estimation, specification, testing, and applications such as forecasting (Adebile and Shangodoyin, 2006; Chikhi and Diebolt, 2009b; Potter, 1999; Umer *et al.*, 2018; Van Dijk *et al.*, 2002; Wahlström, 2004). Smooth transition models, which justify the sources of non-linearity, may be appropriate to provide a privileged framework for the study of asymmetric stock market fluctuations.

For the history and applications of the STAR model to economic and financial time series see, for example, Granger and Teräsvirta (1993) and Teräsvirta (1994) who classify the market into two phases of recession and expansion. Thus, Teräsvirta and Anderson (1992) use the STAR model to predict quarterly OECD industrial production series. Skalin and Teräsvirta (2002) study nonlinearity in the business cycle by using the model and Baum *et al.* (1999) and Liew *et al.* (2004) in real exchange rates. Sarantis (1999) detects nonlinearities in real effective exchange rates for 10 major industrialized countries and evaluates the forecast accuracy of the STAR model over the random walk model. Eitrheim and Teräsvirta (1996) introduce LM tests for the hypothesis of no error autocorrelation, for the hypothesis of no remaining nonlinearity, and that parameter constancy to evaluate the specification of the STAR model. Acemoglu and Scotts (1994) study the relationship between business cycle regimes and nonlinearity in the UK labour market. Öcal (2000) applies the STAR model to test the nonlinearities in growth rates of the UK macroeconomic time series. Escribano and Jordá (2001) investigate the selection of the STAR model by varying some of the parameters and conditions in the models. Wahlström (2004) compares forecasts from the LSTAR model to those from a linear autoregressive model. In turn, Chikhi and Diebolt (2009b) analyse the cyclical behaviour of the German annual aggregate wage earnings by using the LSTAR model. Zhou (2010) evaluates the STAR model in the presence of a structural break in the industrial production index of Sweden. Cuestas, Gil-Alana and Mourelle (2011), Yaya and Shittu (2016), and Yaya (2013) apply the STAR model to the Nigerian inflation series. Tayyab, Tarar, and Riaz (2012) evaluate the suitability of the STAR model specification for real exchange rate Modelling. Adebile and

Shangodoyin (2006) propose an alternative representation of the original version of the logistic STAR model. Meanwhile, Umer *et al.* (2018) compare the performance of STAR and linear AR models by using monthly returns of Turkey and the FTSE travel and leisure index. Aliyev (2019) examines the efficiency of the Turkish stock market by using the STAR model and evaluates its forecasting performance. For a review of threshold time series models in finance, see also Chen *et al.* (2011).

The limitation of these works is that they do not capture the nonlinearity structure in the conditional variance. The assumption of white noise on the LSTAR model residuals ignores the presence of conditional heteroskedasticity; however, the financial series are generally characterized by time-varying volatility that can be modelled by ARCH-type models (Engle, 1982; Bollerslev, 1986) that are often used to study the behaviour of asset returns or innovations of the 'parent' model. Franses *et al.* (1998) and Lundbergh and Terräsvirta (1999, 2000) combine the Smooth Transition Autoregressive (STAR) models (Granger and Teräsvirta, 1993; Teräsvirta, 1994) with GARCH errors (Bollerslev, 1986) and with the Smooth Transition GARCH errors (Gonzalez-Rivera, 1998; Hagerud, 1997). Their results indicate that all models improve upon the linear GARCH(1,1) model and that the STAR-STGARCH model sometimes yields favourable forecasting results. Thus, some authors have used the STGARCH or the STAR-STGARCH to study empirically financial time series. Concerning the STGARCH model, several authors have introduced these specifications (Anderson *et al.*, 1999; Gonzalez-Rivera, 1998; Hagerud, 1997; Medeiros and Veiga, 2009) to model the impact of news on volatility. Lubrano (2001) estimates the STGARCH model using a Bayesian approach. He shows that weekly data of the CAC 40 Paris index did not present non-linearity according to the specification tests, but daily data did. Yaya and Shittu (2016) test nonlinearity and asymmetry in the volatility of bank share prices by using the STGARCH. Their results show that the selection of LSTAR models is affected by the structure of the innovations and this improved as the sample size increased. Regarding the STAR-STGARCH modelling, Chan *et al.* (2002) analyse trends in the development of more ecologically-friendly technologies using the STAR-GARCH model. The regime-switching LSTAR-GARCH model is found to be optimal for modelling the ecological patents ratio. Chan and McAleer (2003) compare algorithms for quasi-maximum likelihood estimation of the STAR-STGARCH model in the presence of extreme observations and outliers using SP500 and Nikkei 225 stock indices. They show that the interpretation of the model can differ according to the choice of algorithm. Reitz and Westerhoff (2007) use the STAR-GARCH model to study the impact of heterogeneous speculators on the commodity market. This model indicates that their influence positively depends on the distance between the commodity price and its long-run equilibrium value. Pavlidis *et al.* (2010) examine the impact of conditional heteroskedasticity and investigate the performance of different heteroskedasticity robust versions. Their simulation indicates that conventional tests can frequently result in finding spurious nonlinearity. Guo and Cao (2011) include asymmetry effects in the transition dynamics of the STGARCH model. Their empirical evidence shows that the model outperforms many existing GARCH specifications in the literature.

Chan and Theoharakis (2011) estimate the m-regimes STAR-GARCH model using quasi-maximum likelihood with parameter transformation. Ben Haj Hamida and Haddou (2014) study exchange-rate dynamics for the Maghreb countries using the STAR-STGARCH model. They indicate that the region's exchange rate follows a non-linear dynamic of ESTAR type for Morocco and LSTAR for Algeria. For Tunisia, the REER is of the LSTGARCH type, which highlights the asymmetric effect of unforeseen shocks on conditional volatility. Midilic (2016) applies the STAR-GARCH model using the Iteratively Weighted Least Squares (IWLS) algorithm to forecast daily US Dollar/Australian Dollar and FTSE Small Cap index returns. Out-of-sample forecast results show that the forecast performance of the STAR-GARCH model improves with the IWLS algorithm and the model performs better than the benchmark model. Livingston and Nur (2018) use the Bayesian inference for the smooth transition autoregressive STAR–GARCH models. Finally, Bildirici *et al.* (2020) suggest the Logistic Smooth Transition Generalised Autoregressive Conditional Heteroskedasticity long-short term memory (LSTARGARCHLSTM) method to analyse the volatilities of WTI, Brent, and Dubai crude oil prices under the influence of the COVID-19 pandemic and the concurrent oil conflict between Russia and Saudi Arabia. A comparison of their approach with the GARCH and LSTARGARCH methods for crude oil price data reveals that their method achieves improved forecasting performance over the others in terms of Root Mean Square Error and Mean Absolute Error in the face of the chaotic structure of oil prices.

Some authors assume that the innovations follow the Normal distribution, which cannot accommodate fat-tailed properties frequently existing in financial time series. Many studies indicate that this problem can lead to inconsistent estimates. The Student's t-distribution and General Error Distribution can be the two most popular alternatives to capture the heavy-tailed returns. In this case, the density function is known and the maximum likelihood estimator of GARCH parameters can be obtained parametrically under regularity conditions (see Gonzalez-Rivera and Drost, 1999; Francq and Zakoian, 2004). However, in most cases, the innovation distribution is unknown and often replaced by a nonparametric estimate, thus the estimation procedure becomes semiparametric (Di and Gangopadhyay, 2014; Mukherjee, 2006; Newey and Steigerwald, 1997; Pagan and Ullah, 1999). This approach assumes a nonparametric form of the density function (Drost and Klaassen, 1997; Engle and Gonzalez-Rivera, 1991) and avoids the inaccuracy of its incorrect specification, and improves the estimation efficiency (Berkes and Horvath, 2004; Gonzalez-Rivera and Drost, 1999).

Our research, in contrast to studies that use parametric distributions, employs a nonparametric maximum likelihood method to estimate semi-parametrically our model. To the best of our knowledge, this model has never been applied to test the weak-form efficiency of ETF markets using semiparametric estimation but existing literature does focus on symmetric and asymmetric GARCH models with parametric distributions (see Narayan *et al.*, 2016 and Aliyev *et al.*, 2020). In addition, some studies treated the weak form efficiency of ETF markets focusing on parametric and nonparametric tests (Gazel, 2020; Ongere and Ngare, 2020; Rompotis, 2011). So, this paper seeks to test weak-form market efficiency and describe the nonlinear dynamics in the Xtrackers CAC 40 that covers the French ETF market with the LSTAR-ANLSTGARCH model using a semiparametric estimation. We thus test the short-term predictability of the traded asset (XCAC) and the weak-form inefficiency of the French ETF market with limited rationality, which emerges arbitrage opportunities. We apply different statistical tests, including BDS, long memory, Hinichbi spectrum, and Tsay tests. After that, we examine the martingale difference hypothesis (MDH) using the automatic portmanteau (AQ) test of Escanciano and Lobato (2009), the Automatic variance ratio (AVR) test of Kim (2009) and the serial correlation test of Deo (2000).

The paper is structured as follows. The next section focuses on the presentation of the LSTAR-ANLSTGARCH model and its semiparametric estimation. Section 3 outlines the daily XCAC price data and discusses its statistical properties. Section 4 is devoted to semiparametric modelling of the daily return series of XCAC; we compare the predictive quality of AR-GARCH, LSTAR-GARCH, and LSTAR-ANLSTGARCH models with that of a random walk. The last section concludes the study by outlining our findings.

1. Methodology: The LSTAR-ANLSTGARCH specification and semiparametric estimation

We consider a logistic smooth transition autoregressive model (with two regimes) with asymmetric nonlinear logistic smooth transition GARCH errors, called LSTAR-ANLSTGARCH (Chan and Mcaleer, 2003) given as:

$$Y_t = \left(\phi_{10} + \sum_{i=1}^{p} \phi_{1i} Y_{t-i}\right) \times \left(1 - G(Y_{t-d}\,; \gamma_{mean}, c_{mean})\right) + \left(\phi_{20} + \sum_{i=1}^{p} \phi_{2i} Y_{t-i}\right) \times G(Y_{t-d}\,; \gamma_{mean}, c_{mean}) + \varepsilon_t \quad (1)$$

with $$\varepsilon_t = u_t \sigma_t, \sigma_t > 0\,, u_t \sim i\,id(0,1) \quad (2)$$

and $$\sigma_t^2 = \left(w_{10} + \sum_{k=1}^{q} \alpha_{1k} \varepsilon_{t-k}^2 + \sum_{j=1}^{r} \beta_{1j} \sigma_{t-j}^2\right) \times (1 - H(\varepsilon_{t-1}\,; \gamma_{vol}, c_{vol},)) + \left(w_{20} + \sum_{k=1}^{q} \alpha_{2k} \varepsilon_{t-k}^2 + \sum_{j=1}^{r} \beta_{2j} \sigma_{t-j}^2\right) \times H(\varepsilon_{t-1}\,; \gamma_{vol}, c_{vol},) \quad (3)$$

where ϕ_{10}, ϕ_{20} are the constants and $\phi_{1i}, \phi_{2i}, i = 1, \ldots, p$ are the autoregressive coefficients of order p. The parameters and the conditions of existence of classical GARCH specification hold for the ANLSTGARCH model, which realizes smooth changing dynamics (Yaya and Shittu, 2016). The logistic form of the two transition functions $G(Y_{t-d};\gamma_{mean},c_{mean})$ and $H(\varepsilon_{t-1};\gamma_{vol},c_{vol})$ causes the nonlinear dynamics in both the conditional mean and the conditional variance equations, given as (Bildirici and Ersin, 2015):

$$G(Y_{t-d};\gamma_{mean},c_{mean}) = \left[1+exp\left(-\gamma_{mean}(Y_{t-d}-c_{mean})\right)\right]^{-1}, \gamma_{mean} > 0 \quad (4)$$
$$H(\varepsilon_{t-1};\gamma_{vol},c_{vol}) = [1+exp(-\gamma_{vol}(\varepsilon_{t-1}-c_{vol})]^{-1}, \gamma_{vol} > 0 \quad (5)$$

To avoid identification problems in both the conditional mean and the conditional variance equations, the slope parameters γ_{mean} and γ_{vol}, which determine the speed of transition function, are strictly positive with $\gamma_{mean} = 1,\ldots,100$. c_{mean} and c_{vol} are the threshold parameters. The two logistic functions $G(Y_{t-d};\gamma_{mean},c_{mean})$ and $H(\varepsilon_{t-1};\gamma_{vol},c_{vol})$ are twice differentiable continuous functions bounded between [0,1] lower and upper bounds for different values of the transition variables Y_{t-d} and ε_{t-1} and their distance to the thresholds c_{mean} and c_{vol} with $d = 1,2,\ldots,p$. Bildirici and Ersin (2015) observe that the transition is relatively slow for low values of the slope parameters γ_{mean} and γ_{vol}, though the transition between regimes speeds up as γ_{mean} and γ_{vol} take larger values. It is noted that the asymmetric nonlinear logistic STGARCH process, developed by Anderson *et al.*(1999) and Nam *et al.*(2002), generalizes the LSTGARCH model introduced by Hagerud (1997) and Gonzalez-Rivera (1998).Then, for positive variance in the ANLSTGARCH model, it is required that $w_{10} > 0, \alpha_{1k} \geq 0, \beta_{1j} \geq 0, w_{10} + w_{20} > 0$, $\alpha_{1k} + \alpha_{2k} > 0$ and $\beta_{1j} + \beta_{2j} > 0$. If $\gamma_{vol} = 0$ the transition function $H(.)$ is equal to 0.5 and hence, the asymmetric nonlinear LSTGARCH model reduces to a single-regime GARCH model.

Financial time series are often characterized by non-Gaussian distributions. Diverse quasi maximum likelihood methods based on many assumptions on the error distribution have been studied in the literature but the true error distribution is unknown. However, the shape parameter of the density function is often incorrect. This leads to estimate non-parametrically the density function (Di and Gangopadhyay, 2014).

Let $\theta = (\theta_{mean},\theta_{vol})$ be the parameter vector of models (1) and (3) where $\theta_{mean} = (\phi_{10},\phi_{20},\phi_{11},..,\phi_{1p},\phi_{21},..,\phi_{2p},\gamma_{mean},c_{mean})'$ is the parameter vector of conditional mean equation and $\theta_{vol} = (\alpha_{11},..,\alpha_{1q},\alpha_{21},..,\alpha_{2q},\beta_{11},..,\beta_{1r},\beta_{21},..,\beta_{2r},w_{10},w_{20},\gamma_{vol},c_{vol})'$ is that of conditional variance equation. The vector θ is a suitable compact set in $R^{2p+2q+2r+8}$. We define the semiparametric kernel density function based on θ (see Di and Gangopadhyay, 2011):

$$\hat{f}(z) = \frac{1}{Th}\sum_{t=1}^{T} K\left(z-u_t(\hat{\theta})\right)/h \quad (6)$$

where $u_t(\hat{\theta}) = \varepsilon_t/\sigma_t(\theta)$. $K(.)$and h represent the kernel and the bandwidth, respectively. The semiparametric likelihood function at θ can be defined as:

$$L(\theta) = \frac{1}{T}\sum_{t=1}^{T} log\left[\frac{1}{\sigma_t(\theta)} f\left(\frac{\varepsilon_t(\theta)}{\sigma_t(\theta)}\right)\right] \quad (7)$$

with any initial estimate of the parameter vector θ, we can apply the two-step estimation procedure and derive a semiparametric estimate of θ, which is given by

$$\hat{\theta}_{SMLE} = \underset{\theta\in\Theta}{argmin}\hat{L}(\theta)$$

If $h \to 0$ and $Th^4 \to \infty$, the initial estimator is $\sqrt{T} -$ consistent and $Th \to \infty$ to insure the consistency of the kernel density estimator (see Härdle, 1990; Di and Gangopadhyay; 2014 for details on the consistency of the kernel density estimate and its derivatives and the asymptotic properties of semiparametric maximum likelihood).

2. Data description and statistical properties

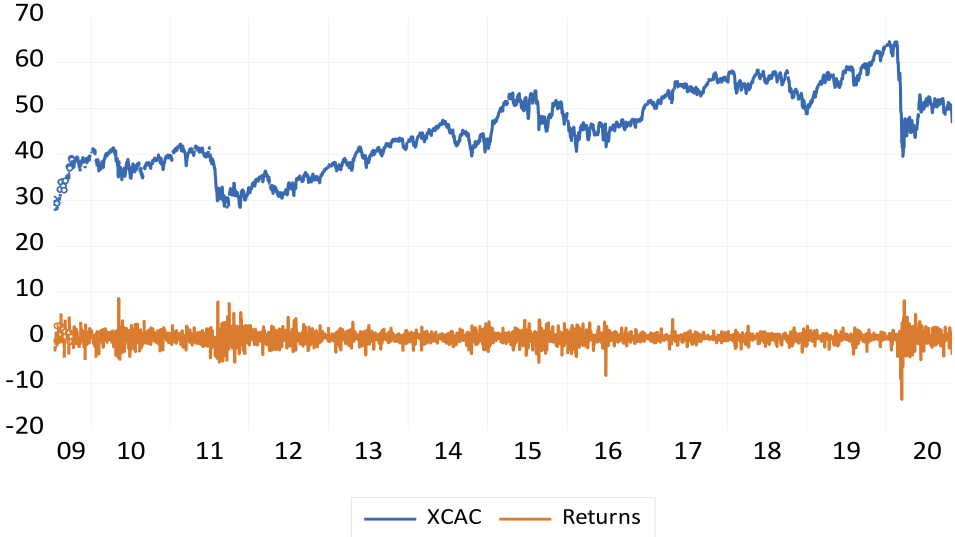
The data used in this paper consists of the daily closing Xtrackers CAC 40 UCITS price that covers the French ETF market downloaded from https://www.investing.com covering a historical period from February 12, 2009, to October 30, 2020, including 2849 observations. To better understand the characteristics of the XCAC40 series, it is necessary to examine some descriptive statistics.

Figure 1 presents time series plots for our studied daily closing Xtrackers CAC 40 UCITS index.The data are transformed into logarithm form. As usual in financial time series, the logarithmic CAC40 series contain a unit root[1]. Our series is therefore differentiated to obtain the daily percentage Xtrackers CAC 40 returns at time t(see also Figure 1):

$$r_t = 100 \times ln(XCAC_t / XCAC_{t-1} \quad)$$

[1]The results of the unit root tests are not reported here but are available on request.

Figure 1. Time series plots for daily French ETF market index and returns



Source: Authors' representation using Eviews 12

The descriptive statistics of the daily XCAC 40 return series in Table 1 reveals that the average return is positive and the French ETF market exhibits a high risk degree as measured by the standard deviation (134.289%). On the other hand, the series is negatively skewed and the data are asymmetric. The value of kurtosis is greater than 3, indicating leptokurtic and more peaked distribution. As known in financial time series, the Jarque-Bera test (Jarque and Bera, 1987) confirms the non-normality of the distribution. The rejection of normality partially reflects the nonlinear dependencies in the moments of returns series. The ARCH-LM test result thus shows that XCAC 40 returns are characterized by the presence of ARCH effect.

Table 1. Summary statistics for daily Xtrackers CAC 40 returns

Mean	Std. Dev. (%)	Skewness	Kurtosis	JB	ARCH(1)
0.0158	134.289	-0.468	10.814	7350.285 (0.000)	41.804 (0.000)

Notes**:** Tests are performed by the authors using Eviews 12 software. (.): The p-Value. We reject the assumption of normality H_0 because the Jarque-Bera (JB) statistic is greater than the critical value of chi-square distribution with 2 degrees of freedom at 1%. Moreover, we reject the homoscedasticity assumption H_0 (there is an ARCH effect in the data because the ARCH-LM statistic is greater than the critical valueofchi-square distribution with 1 degree of freedom at 1%.).
Source: Author's calculations

To test the existence of a nonlinear structure in Xtrackers CAC 40 stock returns and detect the nonlinear behaviour of volatility, we use the Hinichbispectrum test (Hinich and Patterson, 1989) for linearity and Gaussianity and the Tsay test for neglected nonlinearities (Luukkonen *et al.*, 1988; Tiao and Tsay, 1994; Tsay, 2001). Given Table 2, the Gaussianity and the linearity statistics are strictly greater than the critical value of standard normal and that of chi-square distribution at 5%, with two degrees of freedom, respectively. The null hypothesis of linearity and Gaussianity is strongly rejected for returns and volatility. In addition, the Tsay test, which can be considered for LST variant against STAR or TAR, confirms nonlinearity because the F-statistics are greater than the critical value at 5%. We find the presence of a logistic smooth transition in the returns and volatility processes. It is thus essentially due to the large variance change in the time period.

Table 2. Hinich bispectrum and Tsay tests for linearity

Series	Hinich bispectrum test					Tsay test
	Frame Size	Lattice Points	Test Quantile	Linearity	Gaussianity	F_{Tsay}^{4}
Returns	53	169	0.8	654.134 (0.000)	113381.916 (0.000)	5.638 (0.000)
Volatility	53	169	0.8	646.035 (0.000)	111357.455 (0.000)	37.883 (0.000)

Notes**:** Tests are performed by the authors using RATS 9.20 software. The numbers in the table are nonparametric Hinich bispectral test statistics with the null hypothesis H_0 of linearity and Gaussianity, obtaining the chi-squared statistic for testing the significance of individual bispectrum estimates by exploiting its asymptotic distribution. The numbers in the parenthesis are critical probabilities. F_{Tsay}^{4} is the Tsay Ori-F test for neglected non-linearities in an autoregression. We test more specifically against STAR using 4 lags.
Source: Authors' calculations

The BDS statistics presented in Table 3 strongly reject the *i.i.d* assumption, which gives a clear indication of the existence of nonlinear dependencies in XCAC 40 return series for all embedding dimensions. This test leads us to reject the *i.i.d* hypothesis but we do not detect the presence of long-term dependencies. Given this situation, we test the presence of long memory. As it is observed from Table 4, test results for fractional integration show the evidence that the return series exhibits short memory but does not have the behaviour of ARMA. The memory parameter estimated by the Andrews-Guggenberger (Andrews and Guggenberger, 2003), Robinson-Henry (Robinson and Henry, 1998) and the GPH (Geweke and Porter-Hudak, 1983) methods is negative but not significant in all methods. The absence of a long memory indicates that agents can only anticipate their returns to a short time horizon. Indeed, the informational shocks have transitory effects on French ETF returns.

Table 3. BDS test results on the series of Xtrackers CAC40 returns

m	BDS stat.	Prob.
2	10.840	0.000
3	15.255	0.000
4	18.273	0.000
5	20.570	0.000

Notes**:** Tests are performed by the authors using Eviews 12 software. The BDS statistics are calculated by the fraction of pairs method with εequal to 0.7. m represents the embedding dimension. The BDS statistics are strictly greater than the critical value at 5% for all the embedding dimensions.
Source : Authors' calculations

Table 4. Results from the ARFIMA(0,d,0) estimation

	GPH	Robinson-Henry	Andrews-Guggenberger
$\hat{d}$	-0.008	-0.011	-0.126
t-stat.	-0.446	-0.833	-1.268

Notes**:** Tests are performed by the authors using Ox 7.20 and RATS 9.20 softwares. $\hat{d}$ is the estimated Long memory parameter with a power of 0.8.
Source : Authors' calculations

We also present the results of the AVR test, the AQ test and the Deo's test, which is robust to conditional heteroskedasticity. These tests indicate whether the weak form efficiency for the French ETF market is rejected or not – please check meaning . Table 5 summarizes the test statistics. The high degree of predictability and, implicitly, of inefficiency of the French ETF market is observed. The automatic variance ratio test, the automatic portmanteau test and the robustified Box-Pierce test statistics suggest a surprising increase of the degree of inefficiency for all aggregation levels. Over the entire period, the martingale hypothesis, which implies that the best predictor of future values of price, given the current information set, is just the current value of price series, is clearly rejected on the French ETF market at 0.05 significant level. The series is a stationary ergodic non-martingale process[2].Regarding the Deo's test statistic, we note that ETF returns are predictable in the short term and the French ETF market is inefficient. Therefore, we reject the weak-form efficiency for the French ETF market.

[2] For unknown $\mu \in R$ and ∇ is the difference operator $Y_t = Y_t - Y_{t-1}$, the martingale difference hypothesis is defined as : $H_0 : E\ (\nabla Y_t | \Omega_{t-1}) = \mu$ and the alternative hypothesis is that the return series is a stationary ergodic non-martingale:$H_1 : E\ (\nabla Y_t | \Omega_{t-1}) \neq \mu$.

Table 5. Automatic serial correlation test results for daily Xtrackers CAC 40 returns

AVR stat.	AQ stat.	*Q1*	*Q5*	*Q10*	*Q15*	*Q20*	Deo's test stat
-9.609 (0.000)	4.115 (0.042)	4.115 (0.042)	14.526 (0.013)	18.585 (0.046)	27.920 (0.022)	32.681 (0.037)	34.268 (0.024)

Notes: Tests are performed by the authors using R software. (.): The p-Value.
Source: Authors; calculations

3. Estimation and diagnostics tests

In order to test the weak-form efficiency using the nonlinear framework, the modelling of XCAC 40 series could be turned towards smooth transition autoregressive models (Lukkonen *et al.*, 1988; Teräsvirta and Anderson, 1992; Tiao and Tsay, 1994;Teräsvirta, 1994) which could be combined with asymmetric nonlinear smooth transition GARCH errors (Gonzalez-Rivera, 1996; Hagerud, 1997; Chan and McAller, 2003) by using nonparametric maximum likelihood, where the innovation distribution is unknown and replaced by a nonparametric kernel density estimate (Di and Gangopadhyay, 2014; Pagan and Ullah, 1999). In practical terms, we estimate AR, LSTAR jointly with GARCH and ANLSTGARCH models by using the semiparametric approach. Initially, we estimate the model to produce residuals. After that, we use these residuals to estimate the nonparametric kernel likelihood, which will be maximized to obtain the final estimate of the model parameters.

Firstly, we select the lags of linear AR in the conditional mean equation by using the sum of squared residuals and the p-values. In this case, the maximum number of lags is restricted to be three and the largest lag to be considered is five. The second stage is to test linearity against LSTAR or ESTAR nonlinearity and select the optimal transition variable of the conditional mean equation by using minimum p-values of F test statistics of all candidate variables $\{r_{t-1}, r_{t-2}, \ldots, r_{t-5}\}$. In view of Table 6, we find that the sum of squared residuals is minimumfor the variables $\{r_{t-1}, r_{t-2}, r_{t-3}\}$ and the F test rejects the linearity for the delays 1 and 2 in the significance degree of 1%. Either an LSTAR or ESTAR should cause rejection of linearity and rejection of H12. H12 is the appropriate statistic if ESTAR is the main hypothesis of interest. We show that H12 is accepted, but H03 is rejected, which means that the LSTAR model is appropriate.

It is possible that the conditional variance is characterized by a nonlinear structure. The financial prices often exhibit nonlinear heteroscedastic behaviour. For this reason, we first test the GARCH specification against the alternative of ANLSTGARCH. Table 7 shows that the volatility of the Xtrackers CAC 40 return series is adequately captured by the asymmetric nonlinear logistic smooth transition GARCH-type model. The values of the critical probabilities argue in favour of an

ANLSTGARCH model. At this stage, we will study the conditional variance of CAC 40 returns by combining LSTAR model with ANLSTGARCH errors using nonparametric maximum likelihood.

Table 6. Lag selection of linear part and STAR type nonlinearity test results

Delay	STAR type nonlinearity					Lag selection of linear part	
	F-stat.	H01	H02	H03	H12	Selected lags	SSR
1	2.890 (0.034)	1.446 (0.229)	3.250 (0.071)	3.968 (0.046)	2.349 (0.095)	1,2,5	401.032
2	3.590 (0.013)	4.652 (0.031)	1.262 (0.261)	3.324 (0.036)	1.747 (0.186)	1,2,3	400.010
3	2.563 (0.053)	1.197 (0.273)	3.684 (0.055)	2.804 (0.094)	2.441 (0.087)	2,4,5	403.113
4	1.488 (0.215)	0.983 (0.321)	2.802 (0.094)	0.679 (0.409)	1.893 (0.150)	3,4,5	411.235
5	1.635 (0.179)	1.021 (0.312)	3.019 (0.082)	0.864 (0.352)	2.020 (0.132)	1,3,4	409.231

Notes: Tests are performed by the authors using RATS 9.20 and GAUSS 3.2 softwares. SSR : Sum of squared residuals. H01 is a test of the first order interaction terms only. H02 is a test of the second order interaction terms only. H03 is a test of the third order interaction terms only. H12 is a test of the first and second order interactions terms only. (.): The p-Value.
Source: Author's calculations

Table 7. LM test for GARCH against the alternative of ANLSTGARCH

Model	**LM**
GARCH	3.628 (0.056)
ANLSTGARCH	154.087 (0.000)

Notes: (.): The critical probabilities.
Source: Author's calculations

It is well known in the literature that estimating the LSTAR-ANLSTGARCH parameters can be problematic due to computational difficulties using different optimization algorithms, which make it difficult for the Maximum Likelihood Estimator (MLE) to obtain the best model in practice (see Dijk *et al.* 2002; Chan and McAleer, 2003). Unusually, there has been very little research investigating the cause of the numerical difficulties in obtaining the parameter estimates and the number of regimes for LSTAR-ANLSTGARCH. This model has STAR type nonlinearity in both the conditional mean and variance and allows the smooth transitions between the regimes to be governed by a logistic function. Hence, we determine the differentiating characteristics of the volatility of the Xtrackers CAC 40.

Table 8. Estimation results

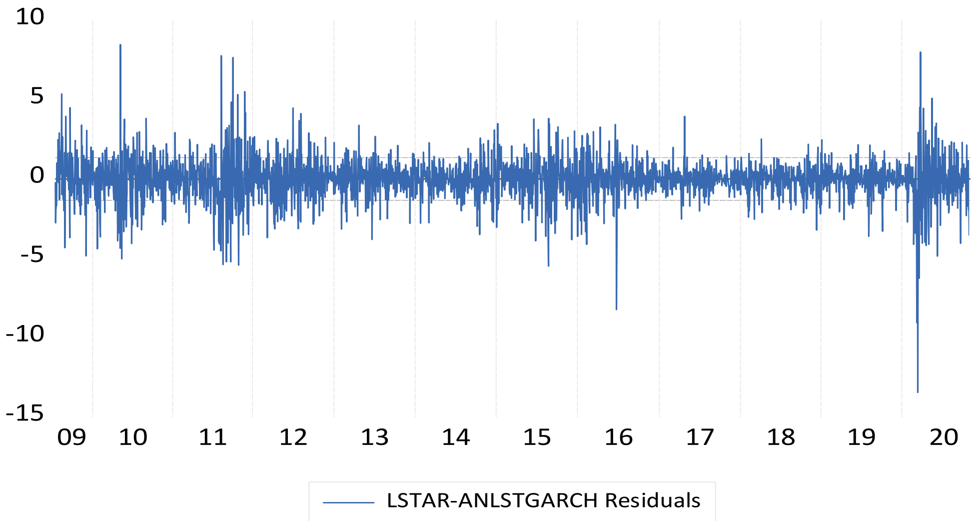
Parameters	AR-GARCH	LSTAR-GARCH	LSTAR-ANLSTGARCH
	Conditional mean		
$\hat{\phi}_{10}$	-	0.0004 (2.572)	-
$\hat{\phi}_{11}$	-0.037 (-1.980)	0.386 (10.170)	-0.031 (-1.12)
$\hat{\phi}_{12}$	-0.024 (-1.262)	0.257 (4.527)	0.936 (20.402)
$\hat{\phi}_{13}$	-	-	0.154 (3.421)
$\hat{\phi}_{20}$	-	-	-
$\hat{\phi}_{21}$	-	-	-
$\hat{\phi}_{22}$	-	0.003 (11.007)	-
$\hat{\phi}_{23}$	-	0.002 (1.947)	0.040 (1.117)
$\hat{\gamma}_{mean}$	-	0.078 (3.421)	0.047 (2.467)
$\hat{c}_{mean}$	-	10.540 (5.895)	0.068 (1.994)
	Conditional variance		
$\hat{w}_{10}$	0.023 (2.792)	0.000004 (15.703)	0.00001 (2.383)
$\hat{\alpha}_{11}$	0.105 (5.338)	0.126 (38.813)	0.077 (7.582)
$\hat{\beta}_{11}$	0.889 (46.46)	0.855 (339.886)	0.944 (62.531)
$\hat{\gamma}_1$	-	-	-
$\hat{\delta}$	-	-	-
$\hat{w}_{20}$	-	-	0.0008 (15.074)
$\hat{\alpha}_{21}$	-	-	0.093 (2.065)
$\hat{\beta}_{21}$	-	-	0.898 (3.283)
$\hat{\gamma}_{vol.}$	-	-	3.877 (9.520)
$\hat{c}_{vol.}$	-	-	-0.006

			(-2.023)
$\hat{\boldsymbol{h}}_{opt}$	0.118	0.118	0.118
L	-4451.486	-4346.271	-4255.148*
Schwarz	3.134	3.074	3.022+
HQ	3.130	3.063	3.011+
ARCH(1)	0.076 [0.781]	0.066 [0.796]	0.265 [0.606]

Notes: Model parameters are estimated by the authors using RATS 9.20 and Ox 7.20 softwares. [.]: The critical probability. The values in parentheses are the Student statistics. $\hat{\phi}_{10}$,$\hat{\phi}_{11}$, $\hat{\phi}_{12}$, $\hat{\phi}_{13}$, $\hat{\phi}_{14}$, $\hat{\phi}_{15}$, $\hat{\phi}_{20}$, $\hat{\phi}_{21}$, $\hat{\phi}_{22}$, $\hat{\phi}_{23}$ are the estimated LSTAR parameters. $\hat{\gamma}_{mean}$ is the estimated rate of transition (LSTAR). $\hat{c}_{mean}$ represents the estimated location variable of LSTAR model. $\hat{w}_{10}$,$\hat{\alpha}_{11}$,$\hat{\beta}_{11}$,$\hat{w}_{20}$,$\hat{\alpha}_{21}$,$\hat{\beta}_{21}$ are the estimated ANLSTGARCH parameters. $\hat{\gamma}_{vol.}$ is the estimated rate of transition (ANLSTGARCH). $\hat{c}_{vol.}$ is the estimated location variable of ANLSTGARCH model. L represents the estimated log-likelihood function. + indicates the optimal Schwarz (SC) and the optimal Hannan-Quinn (HQ). * indicates the maximum log-likelihood function. $\hat{h}_{opt}$ represents the optimal bandwidth for a kernel density estimator.
Source: Authors' calculations

We estimate the AR-GARCH, LSTAR-GARCH and LSTAR-ANLSTGARCH models using a semiparametric version of maximum likelihood based on the Gaussian kernel with an optimal bandwidth. In view of Table 8, we find that the semiparametric log-likelihood function is maximum for the LSTAR-ANLSTGARCH model and most of its coefficients are generally significant. According to Schwarz and HQ information criteria (Schwarz, 1978; Hannan and Quinn, 1979), the LSTAR-ANLSTGARCH model generally outperforms other models. In addition, the smoothness parameter and the threshold value in both the logistic smooth transition autoregressive and asymmetric nonlinear logistic smooth transition GARCH are significantly different from zero. Regarding the estimates of LSTAR-ANLSTGARCH model, the slope and the threshold parameters in both the conditional mean and the conditional variance equations are significant. The results show that the estimated value of the transition speed of regimes generally indicates a rapid change, which means that the switching from recession into expansion is rapid. These results confirm that the conditional variance, which captures the heterogeneous and the volatility clustering is characterized by a nonlinear dynamic with regime switching behaviour. It is also shown that the GARCH parameter of nonlinear part is positive and statistically significant, which implies that positive shocks produce high volatility unlike negative shocks of the same magnitude. The parameters of linear part are positive and statistically significant, which means that the model manages to capture the temporal dependence of the conditional variance. Furthermore, the sum of GARCH parameters in both the linear and the nonlinear part is less than 1. There is still volatility clustering indicating support for asymmetry. Thus, we find that a negative shock increases the conditional volatility more than a positive shock of the same magnitude. In other words, the unexpected shocks have an asymmetric effect on conditional volatility and the speed of adjustment with respect to the equilibrium is faster. On the other hand, the stock price will tend to move to the average price over time and the LSTAR-ANSTGARCH model is stable overall. It should be noted that the residuals of our selected model illustrated in Figure 2, are characterized by the absence of conditional heteroskedasticity: the ARCH-LM statistic is strictly less than the critical value $\chi^2(1)$at 1% for all candidate models.

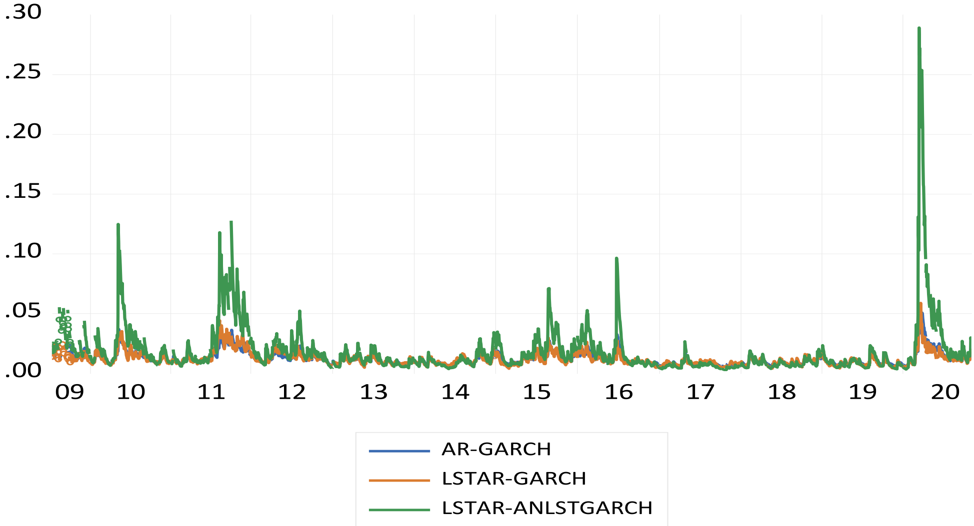
Figure 2. LSTAR-ANLSTGARCH residuals



Source: Authors' representation using Eviews 12

Figure 3 plots estimated volatility of LSTAR-ANLSTGARCH model compared with estimated volatilities of LSTAR-GARCH and AR-GARCH models. We note a higher volatility persistence of ANLSTGARCH. When the level of the true conditional standard deviation changes, the ANLSTGARCH switches from the low-volatility (high-volatility) state to the high-volatility (low-volatility) state, hence the ANLSTGARCH model is more flexible than the GARCH model in accommodating different sizes of shocks. The French ETF market is more volatile in February, 2020, which is when COVID-19 triggered a freefall in the prices of the indices, causing a larger price drop and a bigger increase in variance that converges back towards the "normal" level.

Figure 3. Comparison of estimates of conditional standard deviations



Source: Authors' representation using Eviews 12

4. Forecasts

In order to compare out-of-sample forecast power of the LSTAR-ANLSTGARCHmodel in the French ETF market, we use the mean square error (MSE) and the mean absolute error (MAE), which are defined as:

$$MSE = H^{-1} \sum_{h=1}^{H} (\hat{r}_{T+h} - r_{T+h})^2 \tag{8}$$

$$MAE = H^{-1} \sum_{h=1}^{H} |\hat{r}_{T+h} - r_{T+h}| \tag{9}$$

Table 9 summarizes statistical comparisons of out-of-sample forecasts provided by the AR-GARCH, LSTAR-GARCH, LSTAR-ANLSTGARCH and the random walk models. We find that the LSTAR-ANLSTGARCH model tends to have better predictive results compared to other models in most of forecasting time horizons. Moreover, the three models outperform the random walk model in all forecasting time horizons. However, all the models consider the short-term memory in the conditional mean equation and the conditional volatility, considering that the predictive power for daily XCAC 40 returns reflects the impossibility to forecast up to the longest horizon. The forecast results do not show any sign of efficiency. Thus, the French ETF market does not follow random walk. By plotting the evolution of MSE with forecast time horizons (see Figure 5), we note that the LSTAR-ANLSTGARCH model tends to be better than other models. This is a sign of nonlinearity that we verified with statistical tests.

In order to evaluate the out-of-sample forecast accuracy of LSTAR-ANLSTGARCH over other candidate models, on the one hand, and the random-walk, on the other hand, we can also use the model confidence set (Hansen *et al.*, 2011) to trim the group of models to a subset of equally superior models. The selection procedure begins with the allocation of the initial set of models M_0 to the set $M^*_{95\%}$. In other words, the model confidence set (MCS) function is initiated on all of the candidate models with a confidence level of 95%. If the null hypothesis of equal predictive ability (EPA) is rejected, we remove an inferior model from the group.

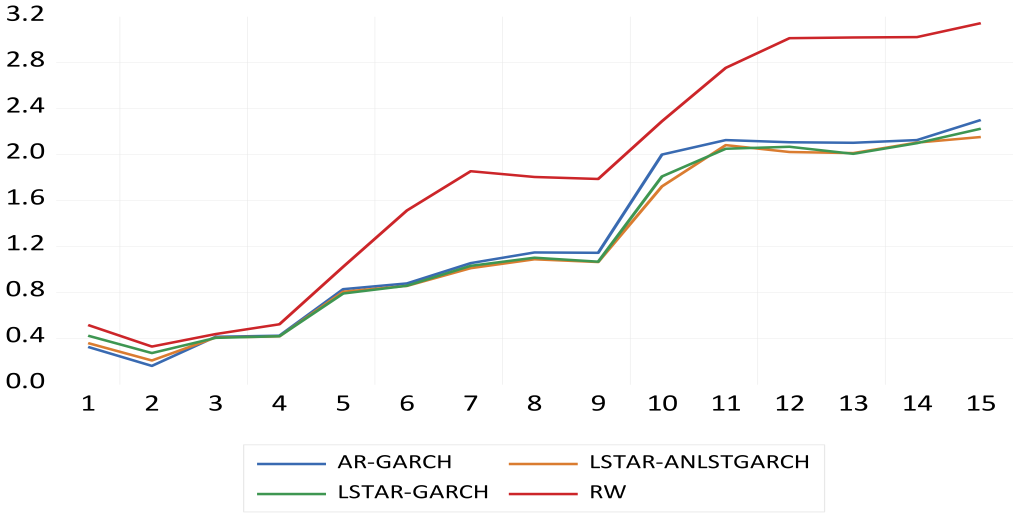
Table 9. Out-of-sample forecast statistics

Function	**Horizon**	**Criteria (10^{-2})**	**AR-GARCH**	**LSTAR-GARCH**	**LSTAR-ANLSTGARCH**	**Random Walk**
Conditional mean (Returns)	1 day	MSE	0.325*	0.424	0.359	0.517
		MAE	0.108*	0.129	0.179	0.253
	2 days	MSE	0.160*	0.272	0.208	0.329
		MAE	0.049*	0.065	0.096	0.118
	5 days	MSE	0.829	0.790	0.807*	1.024
		MAE	0.556	0.545	0.542*	0.892
	10 days	MSE	2.001	1.811	1.724*	2.291
		MAE	0.960	0.920	0.910*	1.147
	15 days	MSE	2.302	2.227	2.154*	3.145
		MAE	1.126	1.125*	1.127	1.312
Conditional variance (Volatility)	1 day	MSE	1.978	1.774	0.673*	-
		MAE	0.444	0.421	0.259*	-
	2 days	MSE	2.670	2.757	0.892*	-
		MAE	0.516	0.525	0.298*	-
	5 days	MSE	0.382*	2.481	2.460	-
		MAE	0.155*	0.355	0.354	-
	10 days	MSE	1.230	1.236	1.229*	-
		MAE	0.241	0.235*	0.236	-
	15 days	MSE	2.495	1.109	1.074*	-
		MAE	0.354	0.222	0.217*	-

*Notes***:** Predictions are calculated by the authors using RATS 9.20 and Ox 7.20 softwares. * indicates the minimum criterion.
Source: Authors' calculations

Table 10 reports a list of models contained in $M^*_{95\%}$. It appears clear that the LSTAR-ANLSTGARCH models are the most consistently chosen by the MCS as the superior models. The results of the MCS selection procedure shows that the LSTAR-ANLSTGARCH model is included in the MCS. The p-values clearly indicate that the null hypothesis of equal accuracy of the LSTAR-ANLSTGARCH model is strongly accepted and this model is included in $M^*_{95\%}$. It is also observed that the LSTAR-ANLSTGARCH model, which incorporates nonlinearity and possible asymmetric shocks, would be the model most likely to be selected and is favoured for modelling XCAC 40 volatility since the p-value is maximum for the LSTAR-ANLSTGARCH model, which creates asymmetrical responses of volatility for both negative and positive shocks. The asymmetry and nonlinearity effects detected on volatility seem to improve the volatility forecasts.

Figure 4. Evolution of MSE criterion with forecast horizons



Source: Authors' representation using Eviews 12

Table 10. MCS and p-values

Model	p-value	Model contained in $\boldsymbol{M^*_{95\%}}$
Random Walk	0.000	LSTAR-ANLSTGARCH
AR-GARCH	0.000	
LSTAR-GARCH	0.048	
LSTAR-ANLSTGARCH	0.845*	

Notes: The p-values that are markedwithone∗indicatemodelsthatareincludedin $M^*_{95\%}$. The MCS is simulated with the R software. Models are calculated by the authors using R software. $M^*_{95\%}$indicates the MCS function, which is initiated on all of the candidate models with a confidence level of 95%.
Source: Authors' calculations

Given that the daily Xtrackers CAC 40 returns are characterized by the presence of nonlinear dynamics in the equations of the mean and by the asymmetric effects in the conditional volatility, the LSTAR-ANLSTGARCH modelling allows computation of better forecasts compared to the other models and the random walk. The returns are short-term predictable. The agents cannot anticipate their returns on a long-time horizon. Indeed, the observed movements appear as the result of asymmetric transitory shocks, which affect the French ETF market. The shock will be persistent in the short term. In addition, the series is characterized by the existence of nonlinearities in the volatility. Consequently, there is an asymmetric impact of positive and negative information on the level of future variance and the weak efficiency assumption of financial markets seems violated for XCAC 40 returns. The investors are able to earn excess return on the basis of some secretly held private, public or historical information.

Conclusions

This study has examined the weak form of efficiency on the French ETF market by using a LSTAR-ANLSTGARCH approach. Firstly, several statistical tests including Hinich bispectrum test, Tsay test for linearity, BDS test, long memory test, automatic variance ratio test, automatic portmanteau test and Deo's test were applied for analysis and results. After the application of these tests, it has been found that Xtrackers CAC 40 is not a weak form of efficient market because its successive return is nonlinearly dependent and does not generate randomly. We also find evidence of threshold behaviour and short memory structure in the returns and volatility series. It is clear that the prices of Xtrackers CAC 40 did not reflect the available market information to all local and international investors who are trading in the French ETF market. Moreover, Xtrackers CAC 40 index is used to make an Exchange Traded Fund that is priced and forecasted, which is important for investors looking forward to make investments on the ETF Market in France due to its mimicking ability.

Secondly, we investigated the presence of nonlinearities in the French ETF returns. In this context, we proposed a semiparametric estimation for LSTAR with ANLSTGARCH errors. We implemented the nonparametric maximum likelihood method to estimate exactly this class of models by taking into account the phenomenon of persistence and nonlinearity for the conditional variance. From the results, informational shocks have transitory effects on volatility and the LSTAR-ANLSTGARCH model shows a superiority over the AR-GARCH, LSTAR-GARCH and the random walk models. By using the model confidence set, the forecasts show a clear improvement compared to the random walk model at all horizons; consequently, the weak-form efficiency of financial markets seems violated for the XCAC 40 returns. Thus, recent works on semiparametric modelling through the ANLSTGARCH process may provide new evidence to better understand the nonlinear dynamics and the asymmetric character of financial series. This semiparametric maximum likelihood estimator is a special case of the general quasi-maximum likelihood in the sense that the parametric form of the density in quasi maximum likelihood is replaced by a consistent kernel density estimate.

The agents have heterogeneous behaviours that vary according to their initial endowments, their individual constraints and their usual activities. In addition, transaction costs are not only variable from one agent to another and based on transaction orders, but they can also define specific thresholds for each investor. The LSTAR-ANLSTGARCH model can reproduce the regime-switching behaviour in the presence of heterogeneous transaction costs and distinct expectations of agents. The smooth transition between regimes can be attributed to the transaction volumes and heterogeneity of investors' expectations.

The current model can be extended in few directions. Firstly, we choose to work with nonparametric distribution in the paper. We could compare our nonparametric approach with other parametric distributions, such as Gaussian, Student or Generalized Error distribution. A more appropriate choice of distribution could improve the performance further under this framework. Secondly, we can study the model by considering some Markov-Switching structure in both the conditional mean and the conditional variance, by taking into account the presence of asymmetric property and switching between different levels of volatility in the LSTAR-ANLSTGARCH. The main limitation of the current model is that it does not capture the long memory structure in the conditional variance. Therefore, we can use daily Xtrackers indices of other European countries that may have a long memory.

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