

# **The VaR comparison of the fresh investment tool- BITCOIN with other conventional investment tools, gold, stock exchange (BIST100) and foreign currencies (EUR/USD VS TRL)**

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## **Abstract**

*In the finance sector, in general, a single VaR method is used for one single portfolio or for all similar portfolios and it hampers the opportunity for comparison. Such shortcoming deriving from trusting one single VaR method results in very incoherent results for the analysis as well as in untrustable transactions based upon those risk estimations. In order to overcome that, similar investments tools/portfolios should be analysed simultaneously by different VaR methods for comparison. Considering such overcome, this study is aimed to compare the VaR (value at risk) estimation methodologies for all 5 separated portfolios (which are similar considering their liquidity and investment process) holding USD, EUR, GOLD, BIST100 Index (Istanbul Stock Exchange Index) and BITCOIN considering their daily return on TRL (Turkish Lira). For performance measurement of different methodologies listed namely as extreme value VaR (GRPD-gnadenko theorem), ewma based volatility filtered historical simulation, historical simulation, delta normal, and bootstrapping; the 3 backtesting procedures and the related statistics are used. The data set is chosen as the period from 02.01.2014 to 20.04.2020 when all returns are recorded daily. The results of such analyses backed with different backtests indicated the different VaR methods performance for the different portfolios. Such results support the idea that, for the similar portfolios, different VaR methodologies and different backtesting process must be applied for the best fit.*

**Keywords:** Historical VaR, Delta Normal VaR, EVT, VaR Backtesting, BITCOIN

## **Introduction**

Since the first official introduction of the risk related measurements of the portfolios, the risk metrics foundation of the VaR (value at risk) concept has been

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used to quantify the risk. Since the beginning of the first introduction of VaR methodology, which was based upon the normal distributions, this methodology has been criticized for its flaws (Dowd, 1998). The very basic methodology, which is called delta normal, is based on the normal distribution. Such distribution, on the other hand, is not followed by most of the financial investment alternatives or portfolios (Rockafellar and Uryasev, 2002; Karahanoglu, 2017). In order to save that VaR analysis from the limitations of the single distribution, the historical simulation is introduced (Pflug, 2000). However, it was then realized that some observations are more important than others which are considered for the historical simulation method. So, different filtered methods were presented, out of which one is based on the idea that the observations must be corrected with the volatility that they have. Based upon that idea, the volatility filtered methods are presented, out of which one is the EWMA method (Trenca and Mutu, 2009). This method is preferred due to its flexibility and due to the fact that it is easy to be applied. But, in order to reach the tail part of the observation distributions and in order to bring them to the VaR analysis, new methods are presented. The ones which are simulation of the observations under distributional limitations (Monte Carlo simulations) and historical observations (bootstrapping) have been started to be applied. Finally, the VaR methodologists are brought some approaches from other disciplines, especially the ones which are concentrated on the observations with less frequent and hazardous effect. The EVT (extreme value theorem) is used to clarify the distributions tails and also to be more into extreme values with less frequency and high magnitude. So, from the chronological perspective; the methods which are used to quantify the risk as VaR are the Delta Normal, Historical Simulation, EWMA modelled Volatility Filtered Historical Simulation, Bootstrapping and Extreme Value Theorem. So there is a hidden problem which method should be used? In fact, where? As the financial products are developed and the number of those alternatives are increased by number, there is a need for finding and choosing the most proper VaR method for those instruments. Such question cannot be answered directly, it needs the process like calculations, comparisons and evaluations.

For considering the Turkey as a developing country, the financial market and the total economy is very fragile against the foreign currency exchange rates. In those countries like Turkey, such currencies are used as investment tool itself to keep the power of the investors wealth in the local economy (Tari and Yıldırım, 2009; Acaravcı and Öztürk, 2002). The Turkish banking system has to calculate and report its currency risk and its currency position to the banking regulatory body regularly. So it is practically and officially extremely important for a Turkish financial institution to quantify its currency risk.

The gold is considered the safe port where the investor would keep its wealth against the crisis and inflations (Topcu *et al.*, 2013). Following the unexpected increase in gold prices, the investors and banks are holding bigger gold portfolio and the demand for it increased sharply. For the Turkish case, the households from the

poorest to richest, all have a small or more gold investments like gold coins or heavy gold portfolios. So gold is a proper investment alternative in personal and institutional level. Quantifying its risks is a relatively important and vital concept. To measure the VaR, Chaithep *et al.* (2012) used EVT and a single backtesting process. Chinghamu *et al.* (2015) did the same but they added ES to EVT methodology and compared results just by one simple backtesting method. Some other scholars used volatility method with GARCH family to estimate the VaR and they have used one single backtesting process (Trück, 2012; Ghorashi and Darabi, 2017; Cheng *et al.*, 2009). The most visible deficiency for such analysis, there are only one or two VaR methodologies were used to estimate VaR for gold portfolio and for backtesting almost just one and the most sample backtesting (VaR exceedance lost number) was employed. Moreover, except for just couple of research, there is not any solid and comprehensive one where the VaR models are compared for the estimation of gold-VaR with other portfolios like currency or stock indices.

The stock market indexes are one of the most important indicators about the local and sometimes the global economy. Moreover, it gives the investor the opportunity to make investment to the local economy. Such indexes are quite good and liquid investment tool (Fisher, 1966). The indexes are seen in most portfolios. Hence their risks should be quantified carefully as well. The Turkish stock market has more than 20 indexes but the most liquid and comprehensive one is BIST 100 index which is composed of the 100 biggest companies whose stocks are transacted in Turkish stock exchange (BIST). For the stock market indexes, general acceptance for the VaR models is to use basic or more advanced volatility filtered models (Basak and Shapiro, 2001; Da Silva and Mendez, 2003; Su and Knowles, 2006; Brooks and Parsand, 2006; Kim *et al.*, 2012). Such model kept its popularity for a single portfolio VaR measurement which includes just stock market indexes within existence of other method preferences (Bali and Çolca, 2004). Specifically, for the BIST 100 index; again the volatility filtered VaR models are more preferred (Bilir, 2016; Unvan, 2020; Eryılmaz, 2015). Some researchers have used different models for comparison (Gunay, 2017; Uyar and Karaman, 2019). Through such works, it is observed that either just one and popular VaR methodology is chosen and the simplest performance measurement process as backtesting (VaR exceedance number) is preferred.

Considering bitcoin as decentralized payment instrument, it has been more than a decade since the world has been introduced with it. As it has a fiscal value against the local currencies, and it is hold by financial institutions and personal investors to increase their wealth against the local currencies, it is also an investment tool (Van Alstyne, 2014). The investors need to see the risks that they are facing with when they hold bitcoin in their portfolios. Just like the other investment alternatives indicated above, the Bitcoin as an electronic currency created by the block chain technology needs to be evaluated from its risks by quantifying. When it comes to the VaR measurement methods, most of the scholars preferred volatility

filtered methods like EJR and GARCH (Stavroyiannis, 2018; Trucios, 2019; Ardia *et al.*, 2019). Moreover, other researchers concentrated on the tail behaviour and used EVT and ES (expected shortfall) (Osterrieder and Lorenz, 2017; Gkillas and Katsiampa, 2019). Just little analysts concentrated on estimating VaR with more than one model. However, like others, they have used either one backtesting or the simplest one as performance measure (Colucci, 2017; Pele and Pele, 2018).

As it is well explained above there are many different investment alternatives and each one of them has its own characteristics and risks. Such risks appear when these investment alternatives are revalued or priced during their holding period. No one can directly say that an instrument goes along with a certain VaR methods well. The main deficiencies in the literature comes from two shortcomings, the lack of researches where more than one VaR methods are applied and compared; secondly the trust of one backtesting method and choosing generally the simplest one. The main target of this study is to apply different VaR methods to different investment alternatives who are similar due to their liquidity and investment procedures. Moreover, in order to make a comprehensive performance comparison, more developed and relatively simple backtesting methods will be applied together.

It is easily seen and shown that only couple of papers concentrate on more than two portfolios simultaneously and applied different VaRs at the same time for comparison. Because of that, in order to fill such a gap, specifically in this research, the different VaR methods and backtesting processes are introduced. Those methods and processes will be applied to 5 different portfolios which will give us the opportunity to see the VaR methods performances from different backtesting perspectives over the period given above.

## 1. The VaR methodologies

The VaR is in fact a single estimation for an expected loose of a single asset or a portfolio. Moreover, for a whole company or firm it is also reported. It is defined as “single value that measures and quantifies the level of financial risk (maximum lost) within a firm, for a portfolio or position over a specific time frame. In other words, it is a potential lost of a portfolio (Dowd,1998):

$$VaR(L) = \inf\{l \in R: \Pr(L > l) \leq 1 - \alpha\}$$

where L is the observation set values and  $\alpha$  is the confidence interval.

Here the 5 main VaR methods will be presented namely “Delta Normal Method”, “Historical Simulations”, “EWMA Filtered Historical Simulations”, “EVT Method”, “Boostrapping”.

### 1.1. Delta Normal

Such modelling is based upon the first term of the famous Taylor Expansion where a function can be expressed with its derivatives as follows, where  $x$  is the observations:

$$f(x) = \sum_{n=0}^{\infty} f^n(a)/n! * (x - a)^n$$

The approach to a function from its first derivative is called as delta. If we just take the first term, the function will be linear and  $a_1$  which is the coefficient of the first term would be called as the Delta ( $\Delta$ ) term of Taylor Expansion:

$$f(x) = a_0 + a_1 * x * f'(x_0)$$

By using the delta coefficient where  $a_1 = \Delta$ :

$$VaR_{\alpha} = -\mu - N^{-1}(\alpha) * \sqrt{\Delta * A * \Delta^T}$$

where

A: is the variance covariance matrix of the variables

$N(x)$  = cumulative normal distribution;  $N^{-1}$  is the inverse cumulative normal distribution which gives us the Z score of the related confidence interval.

$\alpha$  : The Confidence interval of the VaR estimations.

$\mu$  : is the mean value of the observations

And as there will be only one variable in our portfolio, the delta normal formula part becomes;

$$\Delta * A * \Delta^T = \sigma^2$$

Then our VaR can be defined as:

$$VaR_{\alpha} = -\mu - N^{-1}(\alpha) * \sigma$$

where  $\sigma$  is the standard deviation of our single variable (Obradovic *et al.*, 2016).

When we look at the formula; two assumptions make a great noise in our ears, the first one is the linearity which causes not doesn't taking the quadratic or square terms. The second one is the  $N(x)$  term where the normal distribution is accepted. Most of the financial assets return doesn't follow the normal distribution. The early form of VaR suffered from these two assumptions. For the portfolios more than one assets, gamma normal approach is applied and it has brought many advantages (Slustianingish *et al.*, 2019). However, for the single assets the gama normal doesn't bring any advantage. One of the very important points in delta VaR calculations; we need to find the inverse normal distribution of the variable under the level of confidence. That pushes us to be dependent on a parameter. So, delta normal is one of the VaR estimation family member, which is called the parametric methods.

## 1.2. Historical simulations

One of the critics which is directed to Delta Normal is the dependency on normal distribution. In order to save the VaR estimations from such a strong assumption, the historical simulation is presented. The historical simulation is based upon the histogram analysis of the periodical loss and gains. Under the total histogram of the loss and gains (Pritsker, 2006):

$$VaR^{\alpha} = Percentile(\{R_{PF,t+1-n}\}_{n=1}^m, 100 * \alpha)$$

R is the sorted  $m*1$  matrix of the returns (gains and losses);

$\alpha$  has two functions here, at first it determines the confidence interval for VaR and also it helps to find the related loss to calculate the VaR.

So the formula states that the minimum  $100*\alpha$  th observation will resemble the VaR.

Historical simulation believes that in order to see the future for an asset, the past gives all necessary information. None of the assets would betray its past. So it believes that what happens will happen, in order to imagine the future, we need to use past.

The first flaws of the historical simulation are its strong dependency of the past and it limits itself with what happens in the past. It cannot go beyond that border. Moreover, the importance of each observation is the same when VaR estimation is considered. In order to overcome such strong barriers, the filtered and weighted simulation methods are offered (Escanciano and Pei, 2012).

## 1.3. EWMA filtered historical simulations

The relatively more developed and more realistic model is the volatility filtered one which correctifies the observation in accordance with its volatility. If the volatility of an observation series increases, so does the importance of the last observation. As the VaR itself is related to the volatility, the volatility filtered is intuitively logical for estimation of VaR. There are two well developed methods which are EWMA and GARCH estimations. GARCH/ARCH estimations needs to a data set where the error terms of a time series quite fit into ARCH/GARCH family, or they follow GARCH/ARCH process. It is relatively strong estimation and generally it doesn't hold all the time. In our first 100 analyses when we started with GARCH/ARCH filtered method, we saw that almost %17 of the return series followed the ARCH/GARCH (with related time series). So instead of them, the EWMA filtered is preferred.

EWMA (exponentially moving average) is statistical process where the more recent data is considered carefully, as new observations are more important than the older ones. It is a method for estimation of the volatility. The assumption under such methodology is that the volatility of tomorrow is the function of the return of today

and volatility of today ( $f(\sigma_{t+1}) \sim \{\sigma_t, R_t\}$ ). In order to combine them together the model needs a factor which is called decay factor. The model is linear (Ying, 2001):

$$\sigma_{t+1} = k * \sigma_t + (1 - k) * R_t \quad (1)$$

where k: decay factor

$R_t$ : The t-day Return of the data or an asset.

The factor k is called as the decay factor, which is globally accepted between 0.95 and 0.98. In this research we used it as 0.95. The observations need to be revalued with respect to its volatility estimations, so:

$$R'_t = \sigma_t / \sigma_{t-1} * R_t \quad (2)$$

where  $R'_t$  is the corrected or filtered return of the t-day return. So under the case where the volatility increases, so does the value of the observation as well. After all the returns are corrected according to their volatility, the well-known and previously explained historical approach is applied and the related VaR is estimated (Bohdalova and Gregus, 2012).

As it is seen, the EWMA process here is an intermediate process where the returns of a portfolio or an asset are rectified. That method helps researcher to reach the observations beyond its history and it is well independent from the distributions considering the volatility. It can adapt the new market conditions very quickly due to the filter process and volatility consideration. Moreover, it gives importance to the more recent data. That's the reason why for single assets it is used more.

#### 1.4. EVT VaR

Another parametric approach is EVT. That approach concentrates on the tail part of the distribution. It states that, no matter how behaves the data in its central values, the tail behaviour and the structure can be totally different from the center. So it remodels the tail part.

Suppose we have a random variable  $x$ , with an unknown i.i.d function called  $f(x)$  and cumulative distribution function stating as  $F(x) = \Pr(X_i \leq x)$ . The extreme events  $w$  are the ones which are less observable and which can even collapse whole systems; it means a company can lose all of its assets because of them. Such values will be observable at negative side of the distribution.

Let  $\text{Max}_n = \max(X_1, X_2, \dots, X_n)$  be the highest loss in a sample of  $n$  losses. For a sample of i.i.d. observations, the cdf (cumulative distribution function) of  $\text{Max}_n$  is given by the formula:

$$\Pr(\text{Max}_n < x) = \Pr(X_1 < x, X_2 < x, X_3 < x, \dots) = \prod_{i=1}^n F(x) = F^n(x) \quad (3)$$

$F^n(x)$  is estimated by Fisher Tippet theorem. Given that for  $x < x_+$ , where  $x_+$  is the upper end-point of  $F$ , where  $F^n(x)$  goes to 0 logically. As  $n$  goes to  $\infty$ , the asymptotic approximation of  $F_n$  is based on the standardized maximum value is presented as:

$$Z_n = (Max_n - \mu_n) / \sigma_n \tag{4}$$

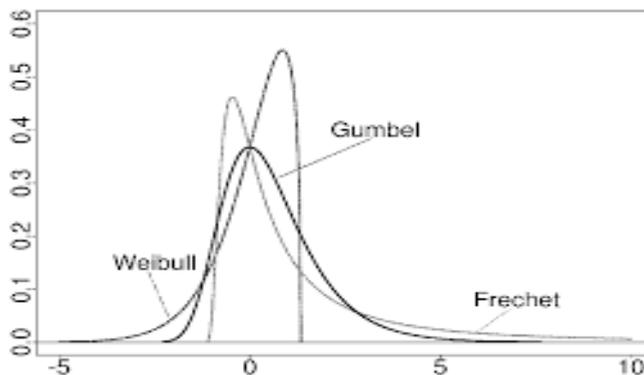
where  $\sigma_n$  and  $\mu_n$  are a scale and location parameters, respectively.

The Fisher-Tippet theorem states if  $Z_n$  converges to some non-degenerate distribution function, this must be a generalized extreme value (GEV):

$$G_\xi = \left\{ \begin{array}{l} \exp(-1 + \xi * z)^{-\frac{1}{\xi}}, \xi \neq 0, 1 + \xi * z < 0 \\ \exp(-\exp(-z)) \quad \xi = 0, -\infty < z < \infty \end{array} \right\} \tag{5}$$

Under such determination, the parameter  $\xi$  is called as shape parameter and it explains the tail behaviour of the distribution function  $G_\xi(z)$ . Under the circumstances where  $Z_n \rightarrow G_\xi(z)$ ;  $Z_n$  becomes the domain function of  $G_\xi(z)$ . If the tail of function  $F$  declines exponentially, it points out thin tailed distribution where  $\xi=0$  and it is called Gumbel Type Distribution. If that tail declines with power function than  $G_\xi(z)$  is called Frechet Type and under such circumstances  $\xi>0$ , and it is fat tailed distribution. The last case is the one where  $\xi<0$ , then  $G_\xi(z)$  is called Weibull distribution and it is thin tailed as well (Dowd, 2007).

**Graph 1. Three main distributions: Gumbel Frechet and Weibull**



Source: Embrechts *et al.* (1999), p. 38

As we indicated before the EVT is more into tail distribution, with another words, the extreme events. So, these extreme events occur or are observed over the certain values, called thresholds. Let the  $u$  be threshold for the distribution function; excess distribution over the threshold  $u$  (conditional probability) can be formulated as (McNeil and Frey, 2000):

$$F(y) = \Pr(X - u \leq y | X > u) = [F(u + y) - F(u)] / [1 - F(u)] \quad (6)$$

For the enough high  $u$  values, there will be function  $\beta(u)$  which satisfies the Fisher Tippet theorem, such that  $F_u(y)$  approximates to generalized pareto distribution:

$$H_{\xi, \beta(u)} = \left\{ 1 - \left( 1 + \xi * \frac{y}{\beta(u)} \right)^{-\frac{1}{\xi}}, \xi \neq 0 \quad 1 - \exp\left(-\frac{y}{\beta(u)}\right), \xi = 0 \right\} \quad (7)$$

where  $\beta(u) > 0$ , and  $y \geq 0$  when  $\xi \geq 0$ , and  $0 \leq y \leq -\beta(u)/\xi$  when  $\xi < 0$ .

EVT-POT (peaks over threshold)/GEV Distribution is mainly dealing with the tail part. Such approach is preferred due to its advantages as it doesn't depend on the tail type distributions. In simple GEV approach where the VaR can be expressed differently is based upon the tail distribution type like Gumbel or Frechet, the determination of the tail type is another problem.

In order to define the tail part, we need  $u$  threshold value. So the function over that  $u$  threshold can be expressed as just like the above one where  $y = x - u$ :

$$F(x) = 1 - \frac{N_u}{n} * \left[ 1 + \xi \left( \frac{x-u}{\beta} \right) \right]^{-1/\xi} \quad (8)$$

By extracting the VaR with confidence interval  $\alpha$  here as:

$$VaR_{\alpha} = u + \frac{\beta}{\xi} \left[ \left( \frac{n}{N_u} * (1 - \alpha) \right)^{-\xi} - 1 \right] \quad (9)$$

By using the R package which is called “fExtremes, the data with 312 observations are fitted and related  $\beta$  and  $\xi$  values are obtained. The function for VaR is created in R as well. This procedure is applied 2000 times by writing a loop. So, 2000 EVT parameters ( $\beta, \xi, N_u$ ) obtained for each variable (BITCOIN, Gold, BIST100 Index, EUR vs TRL and USD vs TRL), and  $2000 * 5 = 10.000$  VaR's are estimated.

## 1.5. Bootstrapping

The bootstrap method can be accepted as resampling technique which is used to estimate basic statistics of a population by sampling a dataset by following the replacement procedure. The method aims to estimate statistics like the mean or standard deviation. The idea which lies behind the bootstrapping is to inference about a population by using the sample data (it is a direct path from sample to population). This process is done by resampling the sample data and making inference about a sample from resampled data (resampled  $\rightarrow$  sample). The true error in a sample statistic is unknown as the population is unknown. This is the place where the trick starts; in bootstrap-resamples, the ‘population’ becomes sample, and this is known; hence the quality of inference of the ‘true’ sample from resampled data becomes measurable.

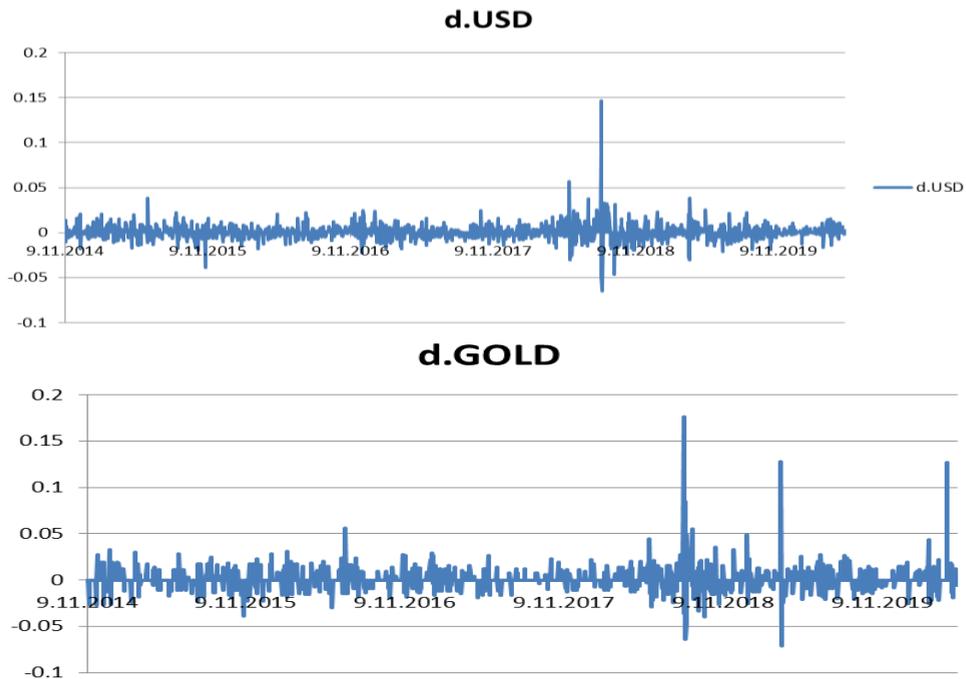
Formally speaking, as first step; let's take the distribution of the original data as  $U$ , and the empirical one which is analogous to interference of the original distribution as  $\tilde{U}$ .  $\tilde{U}$  is the resampled of the original data. The resampled data distribution  $\tilde{U}$  can be assessed because we know  $\tilde{U}$ . Of course this condition is satisfied where  $\tilde{U}$  is a reasonable approximation to original data  $U$ .

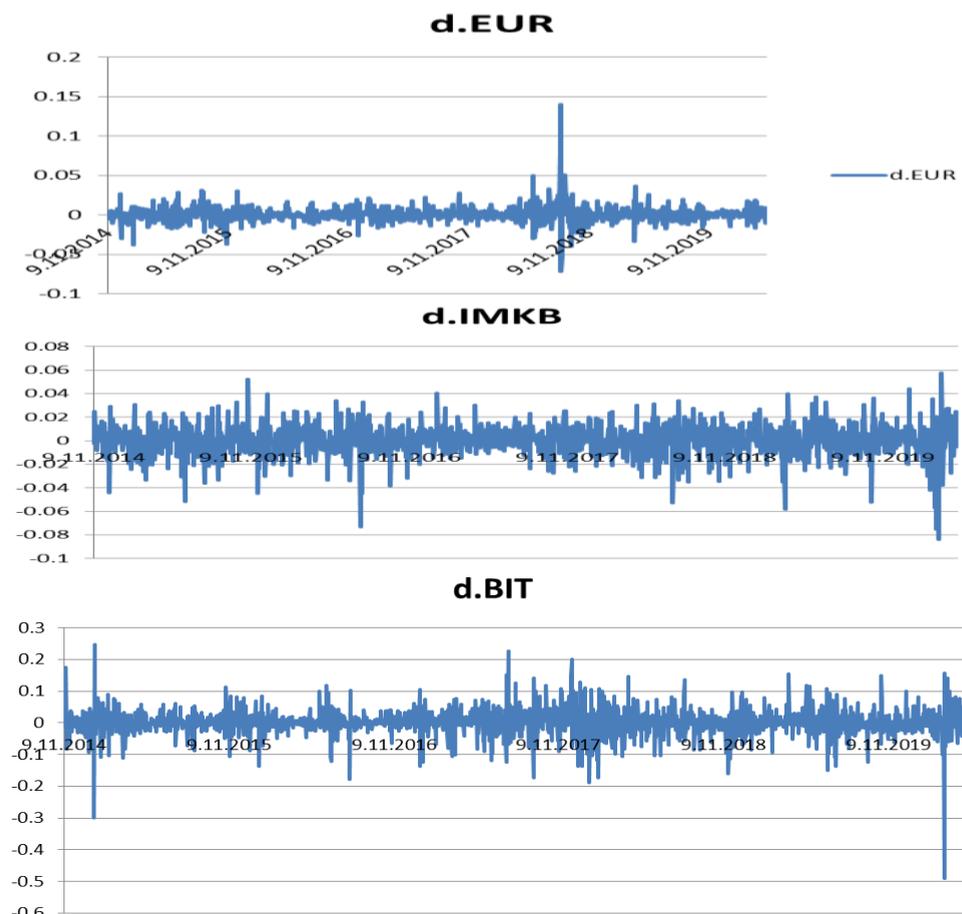
The bootstrapping is applied by following the procedure given as; at first stage the required number of data is taken from the data set as without stressing any restrictions. And followingly, the VaR is calculated from that bucket by ant method including using historical simulation or delta normal. In our analysis we prefer the delta normal, as there is not historical value here. So, for the bootstrapped data, the delta normal is applied. The total number of observation which is taken from the data set kept as half of the whole observations till the day of analysis.

## 2. The Data

The daily return of the related variables against the TRL are summarized as follows:

**Graph 2-6. Daily logarithmic return of USD/GOLD/EUR/IMKB100 and BITCOIN**





Source: author's representation

**Table 1. Main statistics of variables**

Variable	Mean	Std.Dev.	Min.	Max.
USD	0.0005144	0.0078867	-0.06474	0.1470481
EUR	0.0004114	0.0078284	-0.07196	0.1401832
GOLD	0.0006421	0.0099863	-0.07137	0.1766235
IMKB (BIST100)	0.0001803	0.0112265	-0.08416	0.0581040
BITCOIN	0.0015217	0.0591462	-0.49472	0.2485135

Source: author's representation

When we look at the variables, Bitcoin is the one which has far wider range of return than others. Such observation is supported by its standard deviation as well. Moreover, its mean return is higher than others as well. Considering the big volatility

in Turkish Economy happened in 2018 August, USD and EUR maximum return is almost %15.

### 3. The analysis

The data is collected for each variable between 01.02.2014-20.04.2020. For first 311 days, the data is used to create the statistical variables like standard deviation and mean, because these variables are required for the models like EWMA and bootstrapping as well as the other methodologies like historical simulations and delta normal.

There are three methods to measure the performance of the VaR Methods which is used in this research. The first one is the exceedance ratio and related traffic light model. According to the traffic light model offered by Basel Comitee, the observation where the loss is greater than the VaR estimated value for 250-day period are classified as:

1-4 exceedances: Green Light Zone

5-9 exceedances: Yellow Line Zone

More Than 10: Red Zone

In our analysis we used 2000 days, so we just count the number of exceedances and accept it as the total measurement period, so the total exceedance number between 8 and 32 is accepted as green light zone, between 33-72 is yellow line, more than 73 are red zone.

The more complex and accepted method is the Kupiec likelihood method.

As with the recommended standard test, a value-at-risk measure is observed for  $n$  periods, experiencing  $e$  exceedances. The expected exceedance is  $p$  ratio which is  $e/n$ ; the LR statistic is equal to:

$$LR = -2\ln\left(\frac{(1-p)^{n-e} * p^e}{\binom{n-e}{n} * \left(\frac{e}{n}\right)^e}\right) \quad (10)$$

Where the  $LR \sim \chi^2(1,0)$  that is LR follows chi-square distribution. In our case, the observation is 2000 and also the  $p=\%99$ . As Kupiec PF test is chosen for performance, as the PF follows  $\chi^2(0.1)$  the related statistics for %99 is 6.63. Solving the equation for “e” which is equal to 6.63; the limits will be 9.05 and 32.53. The rounded numbers are [9,33]. So the expected exceedance should be between those numbers.

Another test that we applied in our analysis is the Christoffersen’s (1998) independence test which is a likelihood ratio test that looks for unusually frequent consecutive exceedances. In this test statistics again we have to find CRS (christoffersen test statistics) which follows  $\chi^2(1,0)$ . This statistic is based upon the consecutive exceedances;

$a_{00}$  : During 2 consecutive test days, there is not any exceedance

$a_{01}$  : During 2 consecutive test days, first day there is not exceedance but second day there is

$a_{10}$  : During 2 consecutive test days, first day there is exceedance but second day there is not

$a_{11}$  : During 2 consecutive test days, there are exceedances

In order to calculate CRS we need the intermediate step to calculate other variables as:

$$q_0 = \frac{a_{00}}{a_{00} + a_{01}}$$

$$q_1 = \frac{a_{10}}{a_{10} + a_{11}}$$

$$q = \frac{a_{00} + a_{10}}{a_{00} + a_{01} + a_{10} + a_{11}}$$

$$CRS = \left(\frac{q}{q_0}\right)^{a_{00}} * \left(\frac{1-q}{1-q_0}\right)^{a_{01}} * \left(\frac{q}{q_1}\right)^{a_{10}} * \left(\frac{1-q}{1-q_1}\right)^{a_{11}} \quad (10)$$

and  $-2\log(CRS)$  is approximately centrally chi-squared with one degree of freedom. In our cases for the %99 confidence interval the critical value for  $\chi^2(1,0)$  is 6.635. If  $-2\log(CRS) \geq 6.635$  we reject the VaR model.

The process for each methodology is applied and the summary of the results are given below:

**Table 2. Backtesting Results of VaR models for the Portfolios**

Variable	Model	Observation	VaR Exceedance	LR Coverage	Kupiec	Christoffersen Statistics	Test
USD	Delta Normal	2000	35-Yellow Zone	[9.33]*		13.126*	
	Historical	2000	23-Green Zone	[9.33]		9.531*	
	EWMA Filtered	2000	19-Green Zone	[9.33]		6.134	
	EVT	2000	8-Green Zone	[9.33]*		13.129*	
	Bootstrap	2000	17-Green Zone	[9.33]		20.275*	
Variable	Model	Observation	VaR Exceedance	LR Cov	Kupiec	Christoffersen Statistics	Test

EUR	Delta Normal	2000	42-Yellow Zone	[9.33]*	3.224
	Historical	2000	24-Green Zone	[9.33]	5.024
	EWMA Filtered	2000	17-Green Zone	[9.33]	6.998*
	EVT	2000	11-Green Zone	[9.33]	10.472*
	Bootstrap	2000	21-Green Zone	[9.33]	5.392

Variable	Model	Observation	VaR Exceedance	LR Kupiec Cov	Christoffersen Test Statistics
GOLD	Delta Normal	2000	45-Yellow Zone	[9.33]*	0.752
	Historical	2000	26-Green Zone	[9.33]	3.881
	EWMA Filtered	2000	19-Green Zone	[9.33]	1.763
	EVT	2000	10-Green Zone	[9.33]	19.991*
	Bootstrap	2000	44-Yellow Zone	[9.33]*	0.825

Variable	Model	Observation	VaR Exceedance	LR Kupiec Cov	Christoffersen Test Statistics
IMKB (BIST100)	Delta Normal	2000	52-Yellow	[9.33]*	4.402
	Historical	2000	37-Yellow	[9.33]*	3.885
	EWMA Filtered	2000	28-Green	[9.33]	3.338
	EVT	2000	8-Green	[9.33]*	4.491
	Bootstrap	2000	14-Green	[9.33]	2.836

Variable	Model	Observation	VaR Exceedance	LR Kupiec Cov	Christoffersen Test Statistics
BITCOIN	Delta Normal	2000	47-Yellow	[9.33]*	15.731*
	Historical	2000	27-Green	[9.33]	7.773*
	EWMA Filtered	2000	25-Green	[9.33]	8.571*
	EVT	2000	9-Green	[9.33]	12.146*
	Bootstrap	2000	202-Red	[9.34]*	7.036*

Source: author's representation

In the backtesting part, for the exceedance number, LR Kupiec Coverage is calculated as the exceedance between 9-33 the detail of which were given below. For the Christoffersen test, the LR value must be less than 6.635 that was calculated and shown above.

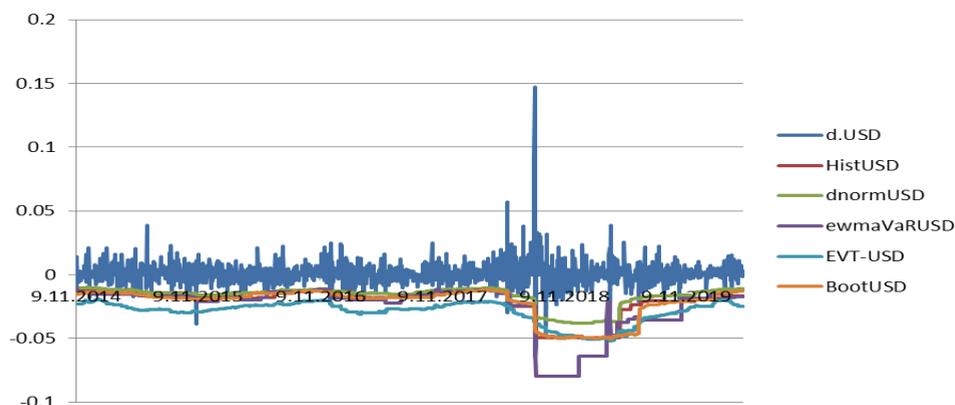
For the USD portfolio, Delta Normal VaR failed from almost all related backtestings, it gives relatively trustable results for VaR exceedance criteria test which results as Yellow. Historical VaR failed from Christoffersen Test where the successive exceedances are valued, EWMA filtered passed from all test. Just like the Historical one, bootstrapping VaR failed from Christoffersen.

Considering the EUR portfolio; the Historical VaR and Bootstrapping VaR passed from all backtests where EWMA and EVT VaRs failed from Christoffersen Test. The Historical VaR again showed the poorest performance by failing two backtestings.

For the GOLD VaRs, Delta Normal VaR and Bootstrapping showed the poorest performance only yellow flag in VaR exceedance criteria. Historical and EWMA VaR passed from all backtests, where EVT VaR failed from Christoffersen Test. When it comes to the BIST100 (IMKB) stock market indices; EWMA and Bootstrapping VaRs showed the most successful performance and passed from all tests. Whereas EVT failed from Kupiec; Delta and Historical VaRs failed from Exceedance Number and Kupiec Tests.

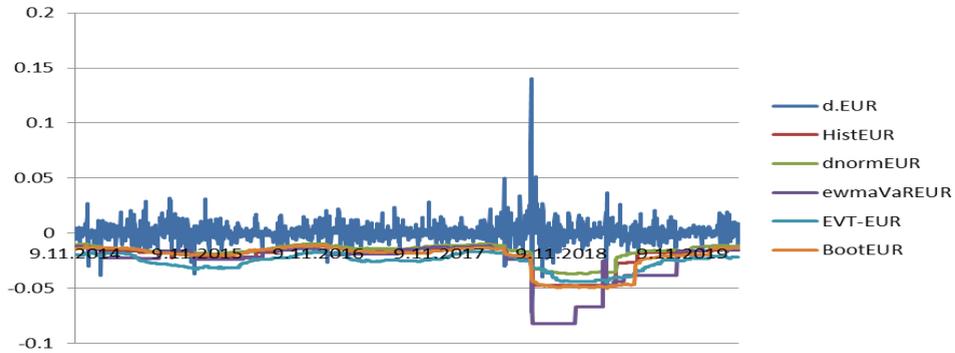
For the BITCOIN portfolio none of the VaR methods could pass from all tests. EWMA, EVT and Historical VaRs failed from just christoffersen test, where other VaRs failed from all tests

### Graph 7. Daily Log Return and VaR Estimations of USD Portfolio



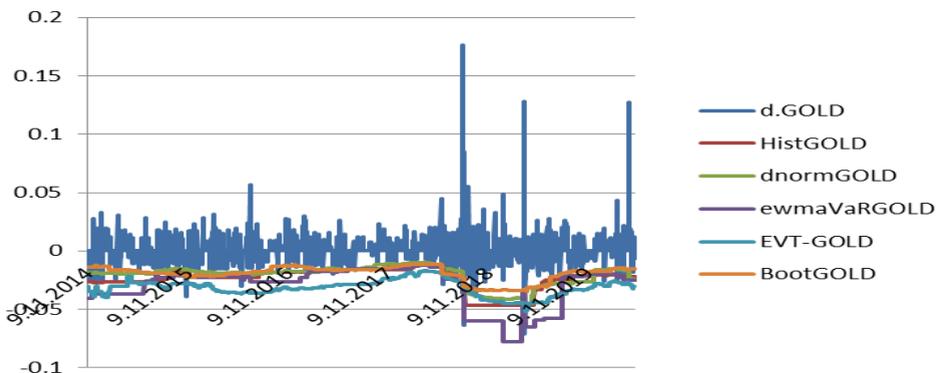
Source: author's representation

**Graph 8. Daily Log Return and VaR Estimations of EUR Portfolio**



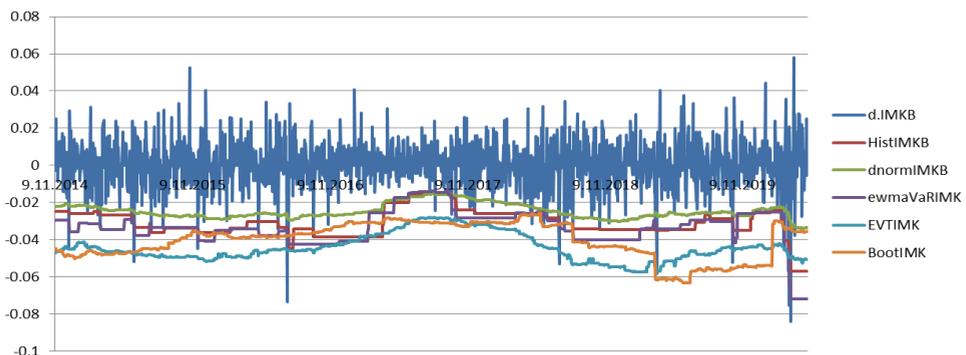
Source: author's representation

**Graph 9. Daily Log Return and VaR Estimations of GOLD Portfolio**



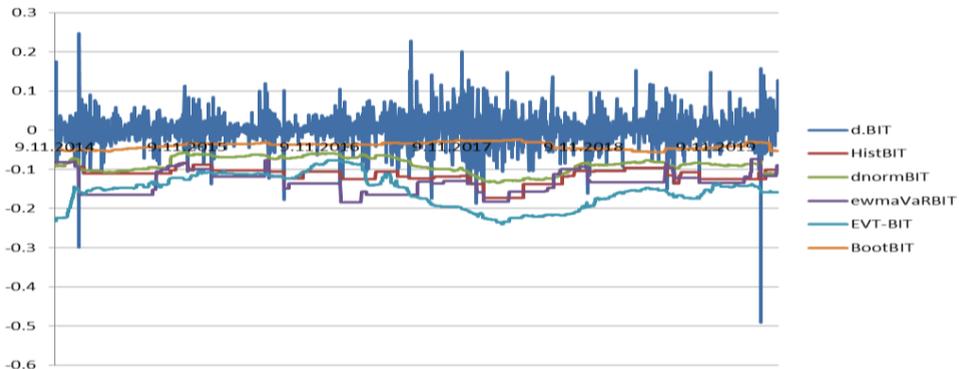
Source: author's representation

**Graph 10. Daily Log Return and VaR Estimations of BIST100(IMKB) Portfolio**



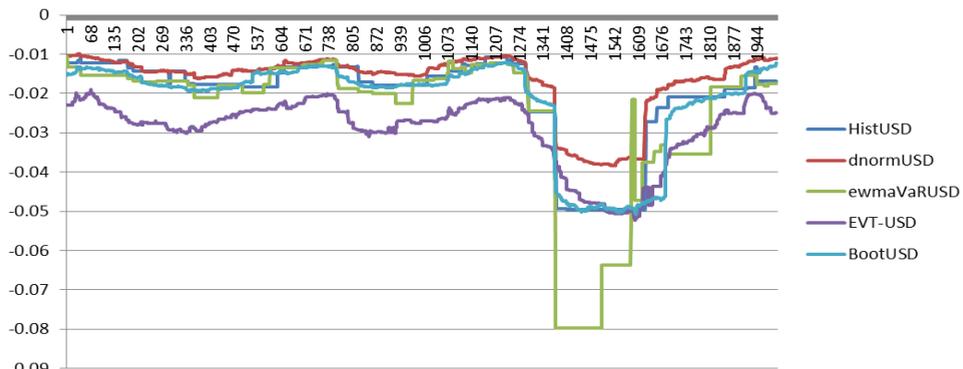
Source: author's representation

**Graph 11. Daily Log Return and VaR Estimations of BITCOIN Portfolio**



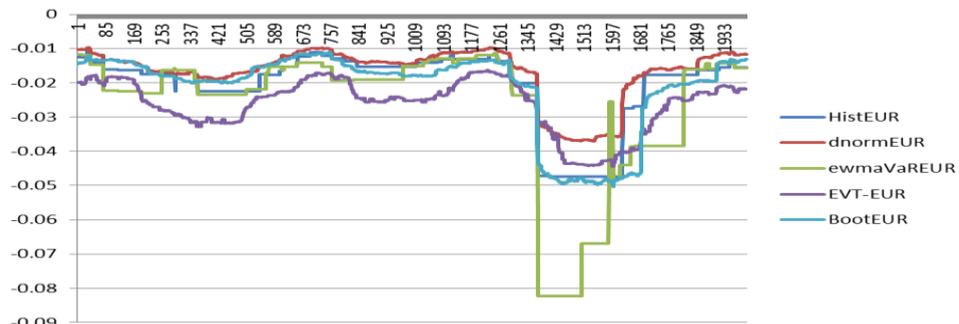
Source: author's representation

**Graph 12. VaR Estimations of USD Portfolio**



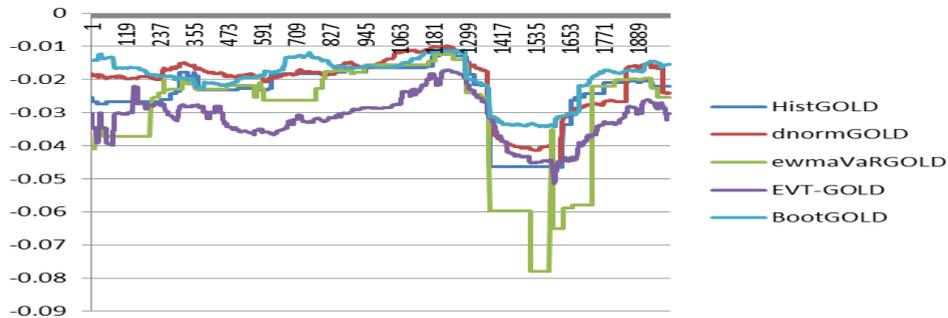
Source: author's representation

**Graph 13. VaR Estimations of EUR Portfolio**



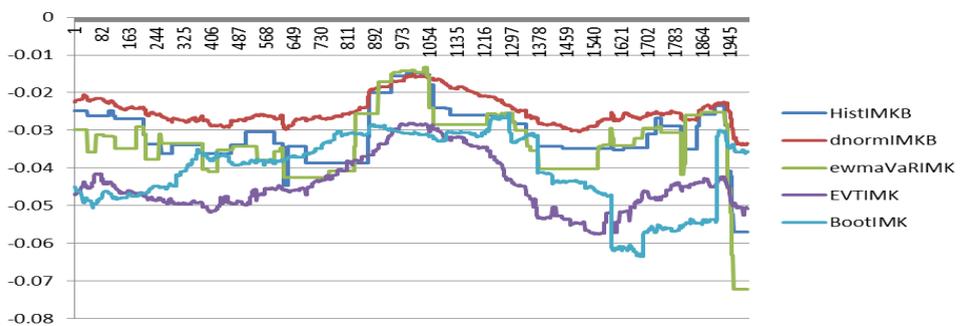
Source: author's representation

**Graph 14. VaR Estimations of GOLD Portfolio**



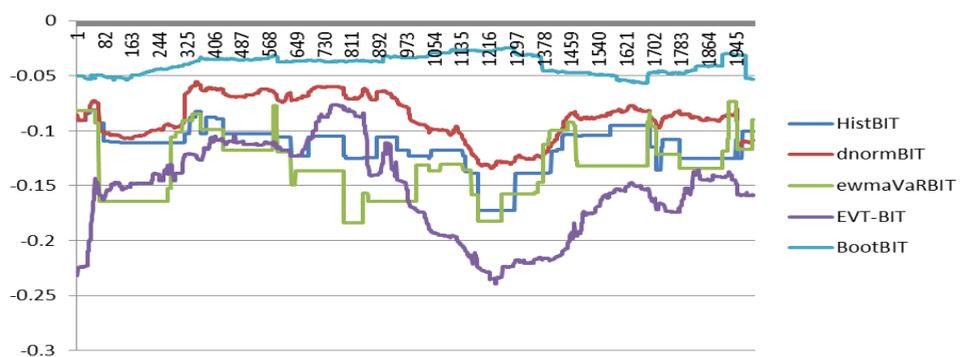
Source: author's representation

**Graph 15. VaR Estimations of BIST100(IMKB) Portfolio**



Source: author's representation

**Graph 16. VaR Estimations of BITCOIN Portfolio**



Source: author's representation

In order to supply clear vision for comparison, the VaR estimation are given repeatedly in the graph with and without daily return of variables. The exceedances can be observed from graph 6-10, whereas the estimations based upon different VaR methodologies can be seen and compared through graph 11-15.

For USD and EUR portfolios which follow almost the same patterns considering the different VaR estimations, EWMA-VaR reacts against the market change most sewer way. It functions like an alert system. All models showed their highest estimations at the same time period. The VaR estimations for the GOLD portfolio follow similar path with EUR and USD where the peak values occurred almost at the same period. EVT and EWMA estimations are generally higher than the other methods. For the BIST100, historical VaR and EWMA Filtered one generally produce the highest VaR estimations which is in fact not too much far from the estimations of other methods. For the BITCOIN all the methods show stable pattern where the VaR estimations are almost 8-10 times higher than the other portfolios. Moreover, EVT VaR for the BITCOIN shows the highest and most unstable estimates.

## Conclusions

When we evaluate the results, for the USD, EWMA Filtered model has passed from all three tests, where Historical and Bootstrap methods passed from 2 tests. Delta normal method failed from all tests. So, the most advised method is the EWMA Filtered whereas the least recommended is delta normal. When we look at the VaR estimations for USD, we would see that, EWMA VaR estimations are not always the highest one however when it feels the danger, it gets higher quickly as August 2018 case. As the danger goes which is measured by volatility, EWMA-VaR estimations goes down quickly. For EUR, historical simulation and bootstrap methods passed from all 3 tests. Delta-normal was only successful at one test. So for EUR, bootstrapping and historical VaRs are the most recommended whereas the delta normal is the least successful one. When we checked the VaR estimation graph of EUR, we see very similar observation with USD. The failure of the EVT is extremely small for both cases. However, this method is very bad at estimation of the consecutive failures. EWMA, in fact coming its own logic, doesn't fall in to that trap.

When GOLD is considered as the investment alternative, it is seen that EWMA and historical VaRs are the best models which passed from all tests; bootstrap is the worst one which was successful at only one test. Such results support the popularity of the volatility filtered VaR methods for GOLD portfolio in the literature. It also submits historical simulation as an alternative. The all three backtesting process supports the results

The method which was the most successful one for quantifying the risks of BIST100 Index is EWMA-VaR method that passed from all tests. EVT and bootstrap passed from 2 tests. Delta normal again the most unsuccessful one. Again here, EVT

gives the least number of failure, however it is really not good at detecting the consecutive failures. Such results are quite similar with the pattern in literature. Just like it, the volatility filtered method is the most successful one. However, for the BIST100, there is no evidence of the success of historical simulation VaR. These results come from the reason that during last 4 years Turkish economy has been in great volatility, which has changed the nature of the BIST100 observations.

Bitcoin is the one where the extremely high and low returns are observed. None of the methods could pass from all tests. However, EWMA and EVT could pass from 2 tests. Moreover, only delta normal method failed from all tests. These results coincide with the general pattern in Bitcoin VaR methodology where volatility filtered and EVT methods are the most popular ones. Moreover, it supplies extra evidence with more robust backtesting procedures and comparison with other methodologies and also it offers the historical method as an alternative one to them. Because of the Turkish economy, the investment alternatives show very volatile return path. That's the reason why the EWMA filtered VaR methodology shows the best performance. As such volatile market would have heavy tail, the normal distribution assumption would not work quite well. That's the reason why delta normal method is the least successful ones among all. EVT method which is in fact very good at VaR exceedance number, it was not as successful as estimating the consecutive exceedances.

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