

Heterogeneous VAT taxation in the Czech economy

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Abstract

This study analyses the VAT tax rate heterogeneity for the case of the Czech Republic. While the European Union recommends tax harmonization, the Czech legislature differentiates among three VAT tax rate groups. Those tax groups' composition has recently changed as the government intends to ease the tax burden during the coronavirus pandemic. Yet, the Czech authority misses an impact evaluation tool of such policy measures for local industries. The EU ambition and the ongoing tax-policy reforms necessitate the model developed in this study which analyses the tax effect at a detailed industrial level. The simulation outcome discloses the sectors which are the most susceptible to VAT taxation changes and suggests the most beneficial tax differentiation scheme to boost economic production. The results support the current tax legislation changes in favour of the more heterogeneous indirect tax rates. A lower tax rate for the industrial sector seems especially advantageous in mitigating the gross domestic product's negative tax impact.

Keywords: computable general equilibrium model, Czech Republic, VAT tax, input-output

Introduction

The value-added tax (VAT) has attracted sizable attention from the European Union. The European Union favours VAT harmonization to promote economic efficiency and the functioning of the internal market. This strategy requires abolishing lower VAT rates and extending national rates (Müllbacher *et al.*, 2013). The ongoing debate requires an in-depth analysis comparing the tax harmonization reform with the tax differentiation's current national tendencies. This study evaluates the economic effects of the uniform and the heterogeneous VAT taxation for the Czech Republic.

The Czech government has also increased interest in the VAT taxation in the presence of the current pandemic. The pandemic's tremendous consequences on local businesses motivated a novel VAT reform with an ambition to ease the burden

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of the COVID affected industries (Act no. 80/2020 Coll., Act no. 256/2019 Coll. and Act no. 299/2020 Coll.). The tax change influences the composition of the three tax groups, taxed by standard rate, 1st reduced rate, and 2nd reduced rate. This ongoing tax reformation necessitates the authority to understand the economic impact of taxing different goods and services.

This article evaluates the VAT harmonization strategy suggested by the European Union and the VAT tax differentiation strategies. The analysis will reveal the Czech industries which are the most vulnerable to VAT taxation and the tax for which industry affects the gross domestic product (GDP) the most. This knowledge can help Czech authorities differentiate the tax rate levied on sectoral products to minimize the economic consequences.

Unfortunately, the tax impact suffers from an endogeneity bias; we struggle to separate an exogenous tax impulse on data. Blanchard and Perotti (2002) offer a Cholesky decomposition within the structural vector autoregressive model as a solution. While useful at a macroeconomic level, this method is hard to apply in a multi-sectoral scenario when the initial tax impulse in one industry (sector) initiates a domino effect on the entire economy.

Modelling the tax multiplier effect requires a multi-sectoral model able to capture industrial interlinkages. This hardship explains the missing research on this topic. The Czech Ministry of Finance but also the Czech National Bank simulate the consumption tax shock with aggregate simulation dynamic stochastic general equilibrium (DSGE) models (see Štork and Závacká, 2010; and Ambriško *et al.*, 2012) which, however, abstract from the industrial differentiation.

Our previous study proves a sectoral heterogeneity unfit for DSGE modelling; the simulation process with hundreds of equations is very time consuming and the results are hard to interpret (Gawthorpe and Safr, 2017). The European Commission understands this difficulty and constructs a deterministic, dynamic general equilibrium model for the input-output analysis (Varga and Veld, 2011). This model's deterministic character assimilates computable general equilibrium (CGE) models and shortens the simulation process. However, the model's dynamic nature, with lags and leads, still hardens the results' interpretability. Several authors find a possible solution by applying computable general equilibrium (CGE) models (see Wing, 2004; Goga, 2009; Mardones, 2015; Taylor 2016). Šafr (2016, 2017), Safr and Sixta (2017), Křístková (2012), Křístková *et al.* 2016 and Kiuila (2015) apply the CGE method on the Czech data.

The CGE model, often static, can be viewed as a simplified DSGE model. The time absence disables the model to capture rigidities, the hump-shape responsiveness of consumption, and feature the New-Keynesian Phillips curve. This reality simplification reduces the model complexity, provides transparent simulation results, and favours the introduction of the new input-output relationships (see Šafr, 2016, 2017). The model can also paradoxically fit data better as it avoids the complicated estimation of abstract values for parameters such as wage and price

rigidity and habit formation. Unfortunately, there is no current research applying this useful method for the Czech tax policy.

The research gap motivates this study to construct and make available an input-output computable general equilibrium (CGE) model for the Czech tax-policy makers. The applied computable general equilibrium (CGE) model is an extended version from Hosoe *et al.* (2010). The model consists of five blocks: household, government, firms, international trade, and market clearing condition. The original version with two sectors is further disaggregated into eight industries and subsequently calibrated on the Czech data. The sectors consist of Agricultural, Financial, Industrial, Energy, Mining, Services, Construction, and Other sectors.

The model benefits from accounting for the tax shock transmission across industries; the shock spreads in the economy from intermediate-input trade. The intermediate-input variable, entering the production process, is an output of one firm that serves as an input in another firm. The input-output analysis is especially suitable for analysing the VAT effects visible at each production and distribution stage. The previous research, without the input-output structure, lacks this multiplication process. The Ministry of Finance study, for example, omits this inter-sectoral flow (Štokr and Závacká, 2010; Aliyev *et al.*, 2014).

Furthermore, besides the insight into the shock transmission through various production stages, the multi-industrial model will reveal the economic effects from taxing different industries. The simulation outcome will show those sectors which are most susceptible to the VAT taxation changes and suggest the most beneficial tax differentiation scheme for economic growth. The findings will help those policymakers questioning the tax harmonization strategy and evaluating the tax heterogeneity to ease the pandemic burden for the Czech economy.

The paper structure will be as follows. The first section outlines the model construction. The second section explains the collected dataset structure and the selected parameters for the model simulations. The third section discusses the simulation findings concerning the differing impact of the VAT taxation on individual sectors, followed by the fourth section which compares the scenarios with the homogenous and heterogeneous tax rates. The conclusion summarizes the simulation results and provides policy recommendations.

1. Model

General Equilibrium models are the common workhorse for policy making utilized by most central authorities around the world. Their attractiveness stems from their rich high-dimensional structure encapsulating economic interlinkages. The system of equations captures the dynamic relationship between economic variables and their mutual dependence. The resulting impulse response functions visualize the consequent propagation of the shocks throughout the system.

In this study, the computable general equilibrium model is an extended version from Hosoe *et al.* (2010). The benchmark model contains five blocks: households, government, firms, international trade and market clearing condition. While the original version assumes only two industries, the new version further disaggregates the model to account for eight industries: agricultural, financial, industrial, energy, mining, services, construction and other sectors.

This detail disaggregation allows to simulate a tax shock for every selected industry, separately. The value-added character of the tax necessitates fiscal-policy models to incorporate a VAT tax shock for every industry. The model captures such taxation structure and allows the tax effect on one industry to transmit to other industries through the intermediate-input trade. In contrast, the aggregate Ministry of Finance and the Czech National Bank models miss the VAT impact spillover effect (see Štokr and Závacká, 2010; Aliyev *et al.*, 2014; and Ambriško *et al.*, 2012).

The VAT tax shock directly affects several model equations. In the model, a household demands a product from a sector that offers the product's relative substitute for the cheapest price, including the VAT tax; it also decides between domestic and imported products, where the domestic ones are subject to the VAT tax while the foreign goods are subject to import tariffs.

The household's consumption choice also depends on the firms' decision-making. Firms are assumed to produce goods based on a constant elasticity of substitution technology that transforms inputs into final products. The production inputs consist of intermediate inputs, capital and labour. Households earn income from supplying the labour and the capital; their income is subject to income tax. The government collects all taxes to finance its government consumption. Finally, the model is closed with market clearing conditions.

The equations below mathematically outline this input-output model.

1.1. Households

A representative household maximizes its utility function U with respect to a good X_i :

$$U = \prod_{i=1}^N X_i^{\alpha_i} \quad (1)$$

where N stands for the total number of goods and α_i reflects the individual good's share in the utility function; subject to the budget constraint:

$$\sum_{i=1}^N P_i^x X_i = \sum_{l=1}^M P_l^f F_l - S^p - T^w. \quad (2)$$

In the above equation, the total expenditure of the household equals the total household income. P_i^x labels the price of a good X_i , P_l^f the price of a factor F_l supplied by the household, S^p the household savings, and T^w the income tax.

The above-stated optimization problem leads to a demand function for the good X_i :

$$X_i^p = \frac{\alpha_i}{p_i^x} (\sum_{l=1}^M P_l^f F_l - S^p - T^w). \quad (3)$$

1.2. Investment

Despite its static character, the model incorporates an investment function, similar to the benchmark version from Hosoe *et al.* (2010):

$$I_i = \frac{\lambda_i}{p_i^q} (S^p + S^g + \varepsilon S^f). \quad (4)$$

The parameter λ_i labels the i -th good's share in the total investment (see the Appendix). The investment originates in the form of household savings, S^p , government savings S^g and current account deficits S^f denominated in a foreign currency with the help of the foreign exchange rate ε (foreign savings).

The households' savings

$$S^p = \sigma_p \sum_{l=1}^M P_l^f F_l \quad (5)$$

represent a share of their earned income and the government savings

$$S^g = \sigma_g (T^w + \sum_{j=1}^G T_j^y + \sum_{j=1}^G T_j^m) \quad (6)$$

are a fraction of the government revenue.

1.3. Firm behaviour

A representative firm in the j -th industry produces goods with three production factors: capital, labour, and intermediate input. The intermediate input represents a product in one industry entering a production process in another one; the input thus connects production processes from multiple sectors. The production process itself consists of two stages (alike Hosoe *et al.*, 2010). In the first stage, the j -th firm demands capital and labour, the inputs $F_{l,j}$, to produce a composite factor (value-added) Y_j . The firm selects such amount of capital and labour to maximize its profit function:

$$\pi_j = P_j^y Y_j - \sum_{l=1}^M P_l^f F_{l,j} \quad (7)$$

subject to the constraint for the composite factor:

$$Y_j = b_j \prod_{l=1}^M F_{l,j}^{\beta_{l,j}} \quad (8)$$

where the applied technology is assumed to be a Cobb-Douglas production function. The parameter b_j is a scaling coefficient in the composite factor production function, and the parameter $\beta_{l,j}$ reflects the share of individual inputs in the production function.

In the second stage, the firm in the j -th industry combines the composite factor Y_j with the intermediate inputs $X_{i,j}$ from the i -th sector to produce a gross domestic output Z_j . The firm decides the inputs' size to maximize its profit:

$$\pi_j^z = P_j^z Z_j - \left(P_j^y Y_j + \sum_{j=1}^G \sum_{i=1}^Q P_i^q X_{i,j} \right) \quad (9)$$

subject to the Leontief production function:

$$Z_j = \min \left(\frac{X_{i,j}}{ax_{i,j}}, \dots, \frac{X_{Q,G}}{ax_{Q,G}}, \frac{Y_j}{ay_j} \right). \quad (10)$$

Maximizing both of these objective functions (equations 7 and 9) results in the optimum intermediate inputs' demand for the j -th firm

$$X_{i,j} = ax_{i,j} Z_j, \quad (11)$$

the optimum amount of the l -th factor that the j -th firm demands:

$$F_{l,j} = \frac{\beta_{l,j} P_j^y}{P_l^f} Y_j, \quad (12)$$

and the optimum composite factor's size:

$$Y_j = ay_j Z_j. \quad (13)$$

Final equations for the firm problem require few rearrangements available in Hosoe *et al.* (2010, p. 91):

$$P_j^z = ay_j P_j^y + \sum_{i=1}^Q ax_{i,j} P_i^q. \quad (14)$$

This last equation defines the price of the gross domestic output for the j -th firm as a function of the composite factor's price P_j^y and the composite good's price P_i^q .

1.4. Government

The government block is essential for our fiscal-policy analysis. Like the benchmark model from Hosoe *et al.* (2010), the government is assumed to levy income tax on households T^w :

$$T^w = \tau^w \sum_{l=1}^M P_l^f F_{l,j}, \quad (15)$$

and the indirect tax on every j -th industry

$$T_j^y = \tau_j^y P_j^z Z_j, \quad (16)$$

where τ_j^y labels the VAT tax rate. Like other general equilibrium models, this model introduces the VAT tax for the final sale (see Aliyev *et al.*, 2014; Hosoe *et al.* 2010). This tax calculation evades the issue of double-counting the tax for multiple production stages since the tax is assessed incrementally. In the Czech Republic, where we tax all added values for a product as well as the initial sale of raw materials, this mathematical representation $\tau_j^y P_j^z Z_j$ is identical to writing the tax for individual production stages. The final collected tax for a product consists of the tax levied on the value-added in the last production stage $\tau_j^y [P_j^z Z_j - (\sum_{j=1}^G \sum_{i=1}^Q P_i^q X_{i,j})]$, where the taxpayer can deduce the intermediate-inputs' tax paid by its supplier; and the tax levied on the intermediate-input sale $\tau_j^y \sum_{j=1}^G \sum_{i=1}^Q P_i^q X_{i,j}$, mathematically:

$$\tau_j^y P_j^z Z_j = \tau_j^y [P_j^z Z_j - (\sum_{j=1}^G \sum_{i=1}^Q P_i^q X_{i,j})] + \tau_j^y \sum_{j=1}^G \sum_{i=1}^Q P_i^q X_{i,j}. \quad (17)$$

Nevertheless, the taxation effect on the sectoral product Z_j , similarly to Hosoe *et al.* (2010) and unlike Aliyev *et al.* (2014), passes through the intermediate-input trade $X_{i,j}$ (equation 9) to other industries. The VAT shock also impacts the output prices P_j^z and, subsequently, the composite factor prices P_j^y and the intermediate-input prices P_i^q (equation 14).

Next to the income and the indirect tax, the government also levies the import tariffs T_i^m

$$T_i^m = \tau_i^m P_i^m M_i. \quad (18)$$

and utilizes the collected tax revenue that exceeds government savings S^g to finance government consumption G_i :

$$G_i = \frac{\mu_i}{P_i^q} (T^w + \sum_{j=1}^G T_j^y + \sum_{j=1}^G T_i^m - S^g) \quad (19)$$

1.5. International trade

The small open character of the Czech Republic allows us to assume an exogenous character of the export prices P_i^x :

$$P_i^x = \varepsilon P_i^{x*} \quad (20)$$

but also import prices P_i^m :

$$P_i^m = \varepsilon P_i^{m*}. \quad (21)$$

The export P_i^{x*} and the import P_i^{m*} prices denominated in the foreign currency are transformed into the domestic prices (P_i^x , P_i^m) with the help of the foreign exchange rate ε .

The following balance of payments constrains the economy:

$$\sum_{i=1}^N P_i^{x*} X_i + S^f = \sum_{i=1}^N P_i^{m*} M_i. \quad (22)$$

The export X_i revenue, denominated in the foreign currency P_i^{x*} , and the current account deficit S^f in terms of foreign currency cannot exceed the import M_i revenue denominated in the foreign currency. Following Hosoe *et al.* (2010, p. 97), the model assumes exogenous dynamics for the current account deficit (or equivalently foreign savings) S^f , but as the authors argue, this variable can be made endogenous if one prefers. The previous authors introduce this variable to enable inequality between the money spent on export and the money spent on import; the exogeneous character then allows them to shock the modelled current account.

The foreign trade allows a firm in the i -th industry to decide between domestic and imported products. The firm selects such domestic/foreign goods' combination that maximizes its profit:

$$\pi_i = P_i^q Q_i - \left((1 + \tau_i^m) P_i^m M_i + P_i^d D_i \right) \quad (23)$$

while producing an i -th Armington composite good Q_i :

$$Q_i = \gamma_i (\delta m_i M_i^{\eta_i} + \delta d_i D_i^{\eta_i})^{\frac{1}{\eta_i}}. \quad (24)$$

The optimization problem respects a constant-elasticity-of-substitution function that results in the optimal demand for the imported goods:

$$M_i = \left[\frac{\gamma_i^{\eta_i} \delta m_i P_i^q}{(1 + \tau_i^m) P_i^m} \right]^{\frac{1}{1 - \eta_i}} Q_i \quad (25)$$

and for the domestic goods:

$$D_i = \left[\frac{\gamma_i^{\eta_i} \delta d_i P_i^q}{P_i^d} \right]^{\frac{1}{1-\eta_i}} Q_i, \quad (26)$$

where the elasticity of substitution parameter η_i is smaller or equal to one. The above demands are functions of the ratio between the relative price for the composite Armington good P_i^q and either the import price P_i^m or the domestic price P_i^d , respectively. The imported goods' demands are also functions of the import tariffs τ_i^m and the input share coefficient in the Armington's function δm_i , while the domestic goods depend on the Armington's share coefficient δd_i ; finally, both functions depend on the scaling coefficient in the Armington composite good's production function γ_i .

A representative firm in the i -th industry also decides how many products to supply abroad and how many to deliver domestically. Such a firm transforms the final product Z_i into products sold domestically D_i and those sold abroad E_i . Hosoe *et al.* (2010) explain the firms' common tendency to customize products for targeted users abroad. The firm considers the transformation function:

$$Z_i = \theta_i (\xi x_i E_i^{\phi_i} + \xi d_i D_i^{\phi_i})^{\frac{1}{\phi_i}} \quad (27)$$

when making the decision which maximizes its profit:

$$\pi_i = (P_i^x E_i + P_i^d D_i) - (1 + \tau_i^y) P_i^z Z_i \quad (28)$$

for the exported goods E_i :

$$E_i = \left[\frac{\theta_i^{\phi_i} \xi x_i (1 + \tau_i^y) P_i^z}{P_i^x} \right]^{\frac{1}{1-\phi_i}} Z_i \quad (29)$$

and for the domestically supplied goods:

$$D_i = \left[\frac{\theta_i^{\phi_i} \xi d_i (1 + \tau_i^y) P_i^z}{P_i^d} \right]^{\frac{1}{1-\phi_i}} Z_i \quad (30)$$

as defined in Hosoe *et al.* (2010, pp. 101-102). The firm's supply abroad depends on the ratio of the export price P_i^x to the aggregate price P_i^z , the indirect tax τ_i^y , the share coefficients of the i -th good transformation ξx_i , the parameter of transformation θ and the elasticity of transformation ϕ exponent. The domestic supply contrasts the export equation with the domestic price variable P_i^d in the denominator and the share coefficient for the domestic supply ξd_i .

The final product Z_i , used as an input in the transformation process of the Armington's commodity, is subject to VAT taxation; firms need to consider this tax when deciding about transforming the good to target domestic or foreign customers (see Hosoe *et al.*, 2010, pp. 101-102). Mathematically speaking, the tax τ_i^y enters the equation 29 from maximizing the equation (28) subject to the equation (29).

1.6. Market clearing conditions

The market clearing conditions secure the equilibrium in the markets. The demand and the supply in the model must equal:

$$Q_i = X_i^p + G_i + I_i + \sum_{j=1}^G X_{i,j} \quad (31)$$

where the Armington composite good Q_i is used by all agents in the model; and the factor market clearing condition is:

$$\sum_{l=1}^M F_{l,j} = F_l. \quad (32)$$

The gross domestic product in this study follows the expenditure method:

$$GDP = X^p + G + I + E - M \quad (33)$$

The input-output character of this model requires data reflecting the flow of the intermediate inputs among industries. The following section describes the dataset.

2. Data

The Czech Statistical Office releasing the input-output tables "SIOT" every five years published the most recent SIOT table in 2015 (www.czso.cz)¹. The above constructed IO CGE model requires aggregation of the table to eight industries: Agricultural (AGR), identified by CZ-NACE 1-3, Mining (MIN, CZ-NACE 5-9), Industrial (IND, CZ-NACE 10-33), Energy (ENE, CZ-NACE 35), Construction (CON, CZ-NACE 41-43), Financial (FIN, CZ-NACE 64-66), Services (SER, CZ-NACE 45-56) and Other industry (OTH, comprising all firms not included in the previous sectors). The below social accounting matrix summarizes the aggregated variables.

¹ Český statistický úřad. (n.d.). Český statistický úřad (retrieved from <https://www.czso.cz/csu/czso/domov>).

Table 1. Social accounting matrix per 10.000 CZK

		Activity								Factor		Indirect tax		Final demand			External Total	
		AGR	MIN	IND	ENE	CON	FIN	SER	OTH	CAP	LAB	IDT	TRF	HOH	GOV	INV	EXT	
Activity	AGR	2,8	0,0	10,3	0,3	0,0	0,0	1,1	0,3					4,3	0,0	1,0	5,8	26,0
	MIN	0,1	0,2	1,5	2,0	0,5	0,0	0,2	0,4					0,5	0,0	0,1	1,7	7,1
	IND	3,2	0,4	79,7	1,7	6,7	0,5	10,8	12,8					20,8	1,6	20,3	258,2	416,8
	ENE	0,3	0,2	6,2	6,5	0,5	0,3	1,9	5,5					12,4	0,0	0,0	3,9	37,7
	CON	0,4	0,1	2,6	0,7	21,3	0,3	2,9	11,1					1,2	0,0	34,4	2,2	77,2
	FIN	0,4	0,0	2,5	0,6	0,7	6,5	3,6	8,3					8,7	0,1	0,3	1,8	33,4
	SER	2,2	0,8	35,9	1,7	3,2	1,3	31,9	13,0					37,8	8,7	7,1	29,5	173,1
	OTH	1,2	0,3	15,6	1,7	11,2	4,2	19,8	44,2					59,2	78,0	22,7	25,7	283,5
Factor	CAP	2,3	1,1	21,9	5,7	3,0	2,8	16,0	44,1									96,9
	LAB	9,7	2,2	76,8	7,6	18,1	12,8	52,2	96,3									275,6
Indirect tax	IDT	-1,3	0,5	14,4	0,7	2,8	2,4	11,7	21,4									52,6
	TRF								0,1									0,2
Final demand	HOH									96,9	275,6							372,5
	GOV											52,6	0,2	27,7				80,5
	INV													199,9	-7,9		-106,2	85,8
External	EXT	4,7	1,2	149,5	8,6	9,3	2,4	20,9	26,1									222,7
Total		26,0	7,1	416,8	37,7	77,2	33,4	173,1	283,5	96,9	275,6	52,6	0,2	372,5	80,5	85,8	222,7	2241,7

Source: author's representation based on data from the Czech Statistical Office data for the SIOT, year 2015

The summary statistics for the selected variables is visible in Table 2.

Table 2. Summary statistics in CZK

	SUM	MIN	MAX	ST.D.
X1j	148678	39	102705	35271
X2j	48198	81	19610	7319
X3j	1159567	4399	796994	267565
X4j	213748	2495	65001	28925
X5j	393760	834	212699	75444
X6j	225956	480	82585	30955
X7j	899745	8347	358776	145425
X8j	980200	2536	441533	147830
Xi1	106069	583	32463	12579
Xi2	21159	201	8347	2677
Xi3	1542648	15220	796994	269025
Xi4	151700	2960	65001	19632
Xi5	440218	490	212699	74697
Xi6	130572	39	65102	24113
Xi7	721964	1838	318857	113429
Xi8	955522	3008	441533	139603
CAP	968777	11266	440697	149251
LAB	2756321	22053	962641	359316
IDT	525534	-12687	214475	82273
TRF	2333	12	884	306

HOH	1449633	4929	592133	206742
GOV	883498	69	779505	271977
INV	858236	-65	344149	133028
EXT	3288297	16904	2582486	884525

Source: author's representation

Table 1 and 2 include the following variables: the intermediate-input flows X_{ij} from the i -th industry to the j -th industry, demand for capital CAP and labor LAB , the collected indirect tax IDT and the transfers TRF , the household HOH and the government demand GOV , the investment INV and the net export EXT .

The sectoral input-output matrix in Table 1 contains the core information for the model calibration. Although the model applies the most recent available dataset, for the year 2015, we can see relative stability of the industrial share over time.

The most essential parameters concern a sector's intermediate-input share in the total production, summarized below.

Table 3. Share of a sector on total production

	2000	2005	2010	2013	2015
AGR	0.03	0.02	0.02	0.02	0.03
MIN	0.01	0.01	0.01	0.01	0.01
IND	0.43	0.47	0.43	0.45	0.38
ENE	0.05	0.04	0.04	0.05	0.04
CON	0.13	0.12	0.12	0.10	0.11
FIN	0.03	0.03	0.03	0.03	0.03
SER	0.14	0.13	0.15	0.15	0.18
OTH	0.19	0.18	0.20	0.19	0.23

Source: author's representation from the Czech Statistical Office data for the SIOT tables

Table 3 documents the highest share of the intermediate inputs from the Industrial, the Other and the Service sectors in the total production. The industrial sector's share reflects the high concentration of the automotive-related manufacturing in the Czech economy. The Czech Republic is among the countries with the highest concentration of automotive-related manufacturing in the world (www.mzv.cz)². Czech firms export a significant share of these automobile components abroad, which is obvious from the high portion of the exported industrial intermediate inputs in Table 2.

Altogether, the model contains over 500 parameters but only two structural parameters, the σ_i and the ψ_i , are calibrated. In accordance with the original study

² Brochure_Czech_Automotive_Industry.pdf. (n.d.). (retrieved from https://www.mzv.cz/file/672401/Brochure_Czech_Automotive_Industry.pdf).

(Hosoe *et al.*, 2010), the value for both parameters equals to 2. The calibration of η_i and ϕ_i parameters also follows the benchmark model where:

$$\eta_i = \frac{\sigma_i^{-1}}{\sigma_i} \quad (34)$$

$$\phi_i = \frac{\psi_i + 1}{\psi_i}. \quad (35)$$

The remaining reduced-form parameters derive from the model steady state, see the Appendix.

Finally, the VAT rate values are exogenously selected to follow an autoregressive process AR(1). The AR(1) parameter is assumed to equal 0.8. The reason for choosing the autoregressive process for the shock is to replicate, with the static model, the prolonged shock impact on the economy that disappears over time. The lengthened shock effect is also apparent in other models; for example, see the Ministry model from Aliyev *et al.* (2014) that also suggests positive autoregressive parameter for tax parameters.

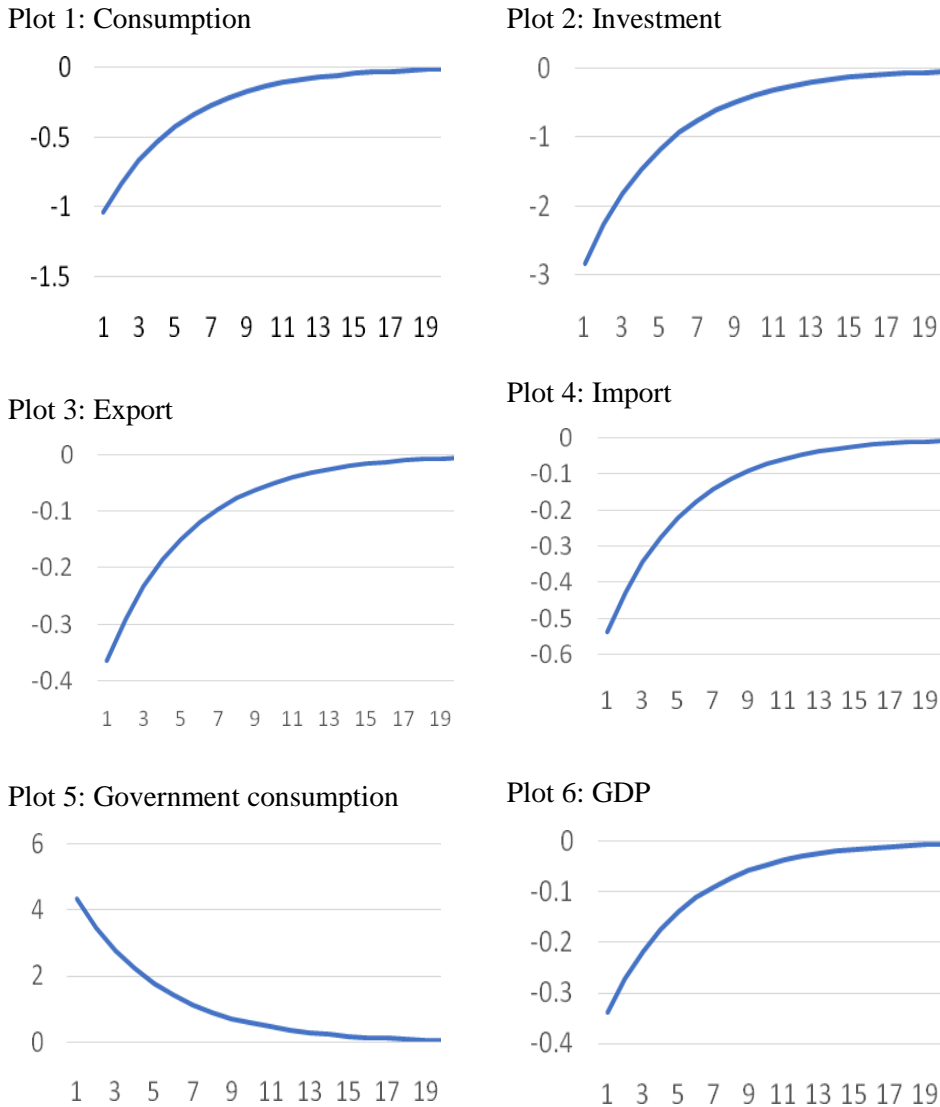
This shock definition for the static model requires iterating the algorithm for every exogenously defined VAT shock value. Specifically, in the first round, the algorithm simulates the model for the first shock value, which is 1; in the second iteration round, for the second shock value equal to 0.8; in the third iteration round, for the third shock value equal to 0.64, and so on, until we obtain 20 simulated periods. As a result, the algorithm simulates variables from every period, independently of each other. This shock's definition allows us to get response dynamics despite the static character of the presented model.

3. Results

This section questions the EU trend of VAT tax rate harmonization and the current Czech tax-policy tendency for the VAT tax rate differentiation. The analysis starts by simulating uniform taxation; the next part reveals the interconnection among individual Czech industries that explains the differentiated industrial response to the VAT tax presented at the end of this section.

The first part analyzes the uniform tax's aggregate economic susceptibility and reveals the tax harmonization's potential economic impacts. These findings will allow us to compare the model validity to the previous fiscal-policy research.

Figure 1. Aggregate impact of the 10 % indirect shock



Source: author's representation

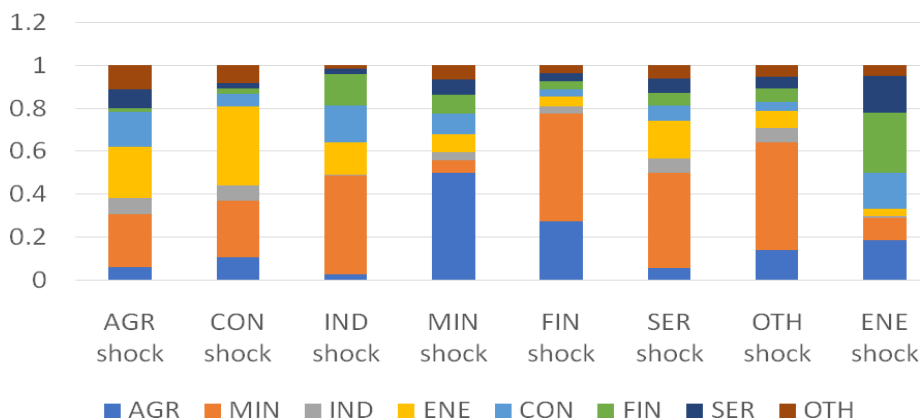
Figure 1 illustrates the VAT shock's economic response, modelled as the 10 percent tax rate increase. The homogenous shock into all industries appears to have an unambiguously negative impact on the Czech GDP. Czech firms downsize their production in the presence of higher production costs. The subsequently lower

profits motivate firms to demand less labour, wages shrink, and so does the consumption expenditure of households for domestic and foreign goods. Lower production leads to lower export. Government consumption seems to be the only GDP component that increases subject to the shock. The government benefits from tax collection growth; this effect translates to higher government spending. This last variable is also why Figure 1 displays the lower GDP reduction relative to consumption or investment drop. The European Commission study (Müllbacher *et al.*, p. 114) also finds a GDP contraction for the Czech economy if goods become subject to a higher VAT tax rate.

Other fiscal studies from the Czech Ministry of Finance, Stork and Zavacka (2010) but also Aliyev *et al.* (2014), Pikhart (2019), and the Ambrisko *et al.*, 2012, measure the GDP susceptibility to 1 pp. VAT tax shock. To compare the previous research with Figure 1, we need to convert the presented shock definition. The CGE model simulates a 10 percent tax shock, given the VAT tax rate value equal to 21 percent in 2015, the ten percent approximately corresponds with 2.1 pp. tax shift. Figure 1 thus illustrates the impact of 2.1 pp. VAT shock; therefore, for 1 pp. change in taxation, the GDP would decline by 0.2 percent. Similarly, Stork and Zavacka (2010), but also Aliyev *et al.* (2014), Pikhart (2019), and Ambrisko *et al.* (2012) find GDP growth drop by 0.2. In sum, the presented model provides comparable GDP responsiveness with the previous research from the Czech central institutions, the Czech Ministry of Finance (Aliyev *et al.*, 2012), and the Czech National Bank (Ambrisko *et al.*, 2012).

This second part questions the inter-connectivity among the Czech industries that affects the economy's tax shock impact. Understanding the structural changes will help interpret the final simulations by illustrating the industrial sensitivity to the VAT shock. The model reflects these sectoral linkages by allowing the shock to pass through intermediate input trade. This part analyses the inter-connectivity by simulating the tax increase for products in one industry and measuring the change for the industry's intermediate-input demand from the other sectors. The figure below shows such an impact of the 10 percent VAT tax shock to one sector.

In Figure 2, a column's label with the suffix "shock" represents the industry subject to the 10 % indirect tax shock. Different colours reflect the industry supplying now fewer inputs to the taxed industry. For example, the orange rectangle in the first column represents the drop of the mining inputs demanded by the shocked agricultural sector. The figure displays that, on the average, taxed industries require the least of the mining and the agricultural products.

Figure 2. Intermediate-input demand from other industries

Source: author's representation

The mathematical intuition for this outcome originates in the model equation:

$$X_{i,j} = ax_{i,j}Z_j. \quad (36)$$

The intermediate-input requirement parameter $ax_{i,j}$ moderates the link between a composite-factor product and an intermediate-input demand. The parameter describes the size of intermediate inputs an industry requires for its production from other sectors.

Table 4. Parameter $ax_{i,j}$ for individual industries

Other industries	0,11
Industrial sector	0,11
Service sector	0,10
Construction sector	0,06
Energy sector	0,05
Financial sector	0,04
Agricultural sector	0,02
Mining sector	0,02

Source: author's representation

The more an intermediate input is required by an industry, the less is the input susceptible to the VAT shock, see the table 5.

Table 5. Correlation between intermediate-input demand and input requirement ratio

Agricultural sector	-0,69382337
Construction sector	-0,03859754
Industrial sector	-0,93385671
Mining sector	-0,94310008
Financial sector	-0,7345093
Service sector	-0,95886719
Other sector	-0,70837072

Source: author's representation

Table 5 provides the correlation coefficients between the parameter $ax_{i,j}$ and an intermediate-input demand change subject to the tax shock. The above table demonstrates a close negative correlation between these two measures. The more an industry requires an intermediate input for production, the less it reduces its demand when reducing production. The only loose correlation appears for the Construction sector. This industry demands a tiny portion of an intermediate input from the agricultural sector relative to its demand from other sectors, around 0.08 percent (visible in Figure 2). This outlier rather distorts the correlation coefficient that otherwise equals 0.76 percent. Overall, the negative correlation in Table 5 reflects the low industrial price-demand elasticity for necessary intermediate inputs and high for the unnecessary inputs.

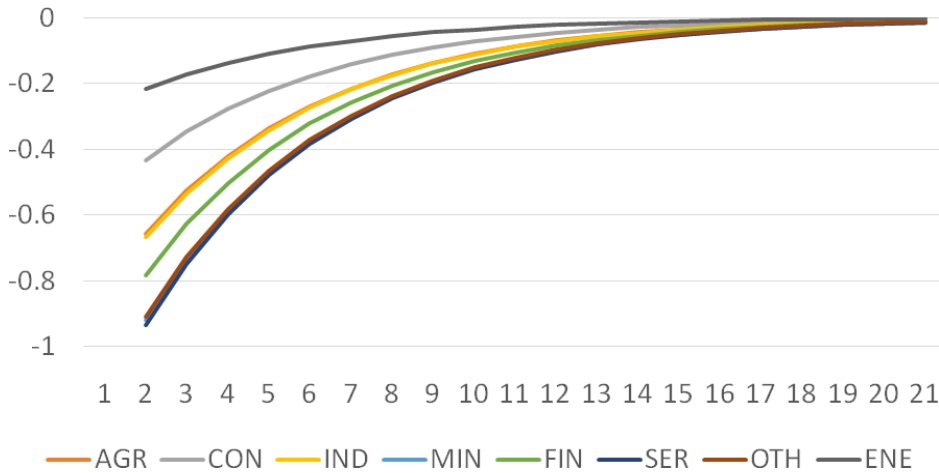
In sum, the demand for the Mining and the Agricultural intermediate inputs is the most sensitive to the adverse tax shock (see Figure 2) because these inputs are required the least by the remaining sectors (see Table 4). This sizable demand drop suggests that the Mining and the Agricultural industries are particularly sensitive to taxation. Policymakers need to consider such sensitivity.

So far, we have looked at the production spillover from the value-added tax; now, we analyse the tax implications for the GDP components for a taxed industry. The figures below illustrate the effect of the 10 percent VAT tax shock on an affected industry's GDP components.

The VAT tax directly affects prices. This price change results in a drop in demand for the taxed good; the reduction's size depends on the good's demand elasticity, as explained above for intermediate inputs. The demand elasticity explains the lower susceptibility of essential (necessary) goods, such as food or electricity, to the VAT shifts (Directorate General Taxation and Customs Union, European Commission, 2008, p. 43). Figure 3 reflects this pattern; the demand for goods from the Energy or the Agricultural sectors belongs among the three least sensitive. In contrast, services such as accommodation or financial services decrease the most. This outcome assimilates the study. Consequently, levying a lower tax rate on the more demand-elastic goods would mitigate the consumption distortions and increase

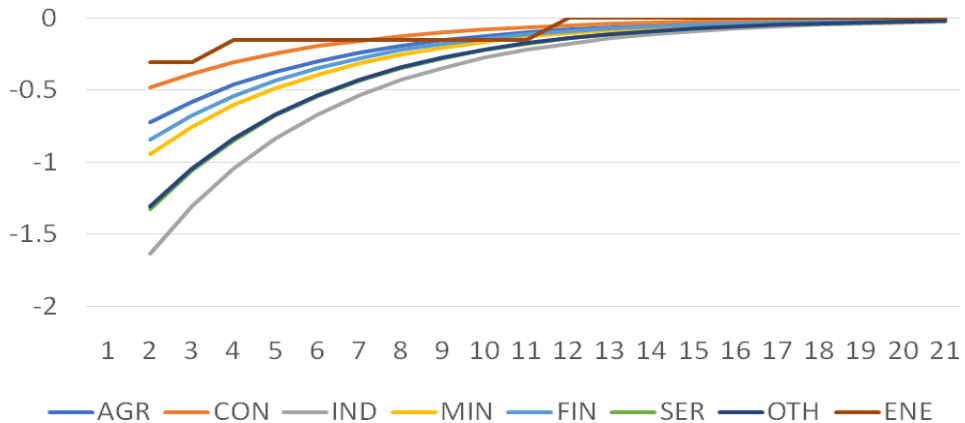
consumers' welfare. However, this analysis abstracts from a governmental policy targeting lower-income households; such a policy could prefer a lower tax rate for the less demand elastic necessary goods.

Figure 3. Consumption



Source: author's representation

Figure 4. Investment

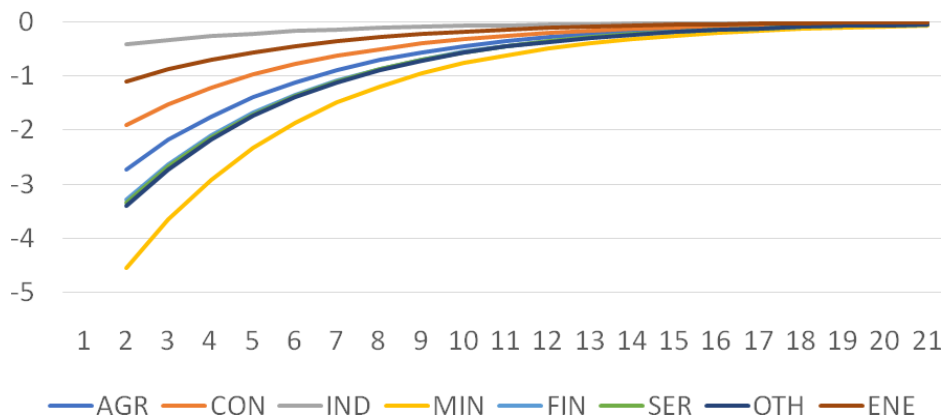


Source: author's representation

The investment from the Industrial and Service sectors contracts the most when subjected to the VAT tax. This differentiated VAT tax effect on the investment

suggests a potential gain for the Czech investment activity from heterogeneous taxation.

Figure 5. Net export



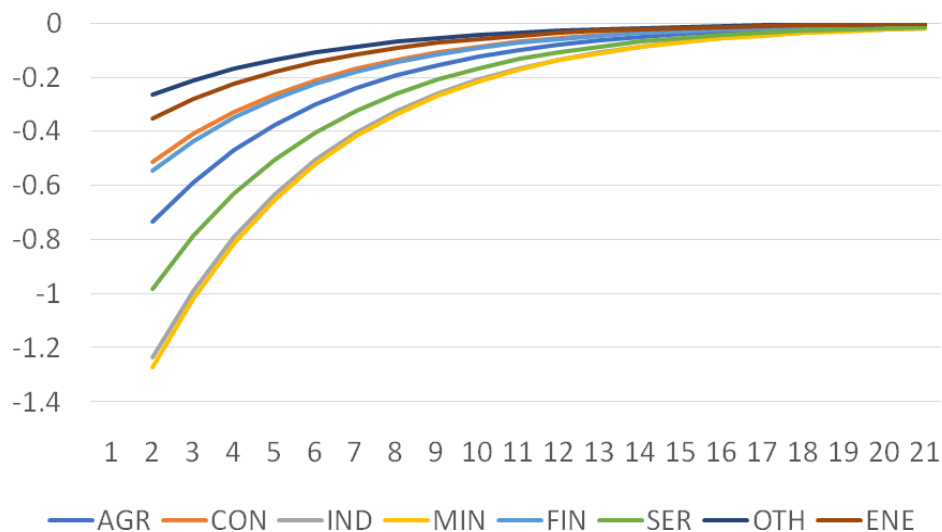
Source: author's representation

Figure 5 illustrates the net export VAT tax susceptibility for the selected industries. This simulation outcome is significant for the Czech economy, which is a small open economy with a high foreign trade share, i.e. around two-thirds of the Czech GDP (www.czso.cz).

The net export drops the most for the same sectors whose intermediate-input supply dropped the most subject to the tax shock - the Mining and the Agricultural sectors (see Figure 2). We can explain this outcome from equation 26, which shows the positive direct relationship between sectoral production and net export. Similarly, the Service sector's domestic production variable drops in response to the shock, which directly affects the net export variable in equation 26 and explains the visible net export decline for the Service sector in Figure 5, despite the service's non-tradable character.

Finally, the low net export sensitivity for the Industrial sector reflects the inelastic demand for the Czech automotive components from abroad. As discussed earlier, The Czech economy hosts one of the highest concentrations of automotive-related manufacturing in the world; the automotive intermediate inputs are thus crucial for the foreign automobile industries (www.mzv.cz).

While the above analysis reveals the tax distortions concerning consumers' and firms' expenditures, we now analyse the sectoral differentiated tax effects on production.

Figure 6. Production responsiveness to the tax shock

Source: author's representation

Figure 6 illustrates a production shrinkage for all the classified industries subject to the indirect tax shock. The Mining, the Industrial, and the Service sectors are those most affected by the shock.

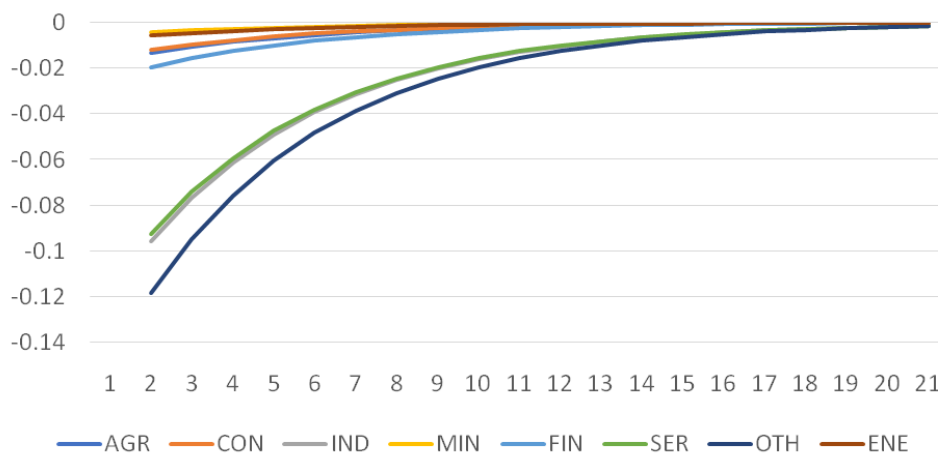
Bouakez *et al.* explain the Industrial and the Service sectors' high sensitivity to price increases by nominal frictions in these sectors. The monopolistically competitive Service sector sizably reduces production in the price-sticky environment; in the durable manufacturing sector, the reaction is similar due to monopoly power. Previous studies, Gawthorpe (2019) and Gawthorpe and Safr (2017), also support the Czech industrial sector's high susceptibility to price shocks. The finding concerning the high Service sector sensitivity supports the recent trend in the Czech Republic to levy a lower indirect tax rate on multiple goods from this sector.

Figure 6 shows the Mining sector to be the most sensitive one to the shock. This outcome supports the earlier hypothesis about a close but negative relationship between the input-requirement ratio and the size of intermediate-input demand change subject to a tax shock. In other words, the lowest necessity of the intermediate-input from the Mining industry leads firms to reduce demand for the Mining products the most in times of production shrinkage (see Table 5). In turn, the Mining sector suffers significantly subject to the tax. This higher susceptibility of the Mining sector towards the VAT tax shock contrast our previous study (Gawthorpe and Safr, 2017). However, while Gawthorpe and Safr (2017) measure

the impact of the price shock on aggregate production, Figure 6 only evaluates the effect at the sectoral level. The low representation of the Mining production in the total output explains the low final Mining effect on the Czech GDP. The figure below respects this intuition.

Figure 7 illustrates the impact of taxing an industry on the aggregate Czech GDP growth. Policymakers interested in the overall economic performance should consider a VAT tax policy respecting the differentiated effect on the sectors.

Figure 7. Aggregate impact of the 10 % shock into an individual sector on the aggregate GDP (%)



Source: author's representation

This total macroeconomic effect from shocking a sector depends on the industry susceptibility to the shock, the industry's production size in the economy, and the shock pass-through to other industries. The model incorporates all these three components, a production function for individual sectors, the intermediate-input requirement for every sector, and a sector's share in the economy. According to Figure 7, the Other, Industrial, and Service sectors represent the three most affected sectors and, together with the Financial and Agricultural sectors, belong to the top five most susceptible to tax changes. The high sensitivity of the Industrial, Financial, and Agricultural sectors corresponds with our previous findings (Gawthorpe and Safr, 2017).

This outcome from shocking an industry on the GDP reflects the respective industrial share in the economy. The Other sector bears the highest share, followed by the Industrial and the Service sectors (see Table 3). The Mining sector's low share explains the low impact of taxing this sector on the GDP despite the sectoral susceptibility to this shock in Figure 6. Next, the Service and Other sectors belong

among the three sectors with the most sensitive sectoral GDP components in Figures 3, 4, and 5. The Service sector's sensitivity is also a result of the high price elasticity for services that households view as non-essential. Finally, the effect from taxing the Industrial sector respects the industrial production drop in Figure 6; mapping a significant portion in the economy, this sector's production significantly affects the Czech economic performance.

The significant differentiation in the sectors' susceptibility suggests potential sizable economic distortions for a homogenous tax system. In other words, an efficient VAT tax system should respect these outcomes. While this section provided a complex analysis for the sectoral heterogeneity, policymakers could consider the simulation outcomes in respecting their policy objectives. For example, the consumption effect could interest policymakers targeting consumer's welfare while targeting the profitability of the Czech companies and finally, the total macroeconomic performance. The next section calculates the difference in the GDP impact from the homogenous and the heterogeneous tax systems.

4. Homogenous or heterogeneous taxation

This section compares a homogenous VAT rate structure with a heterogeneous one. The model simulation results provide the most effective heterogeneous taxation to minimize the GDP growth impact. The homogenous VAT tax rate stays the same as in the previous section, equal to 10 percent (equivalent to the 2.1 pp. increase of the current standard VAT rate). The average of the differentiated tax rates across individual sectors also equals 10 percent.

Table 6 summarizes the simulation outcome either for the even ten percent tax increase or for the tax increase differentiated across the sectors.

Table 6. Simulation outcome

	AGR	CON	IND	MIN	FIN	SER	OTH	ENE	SUM
Shock	9.235	10.275	1.259	28.771	6.160	1.301	1.019	21.980	
GDP change	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.097
Shock	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	
GDP change	-0.013	-0.012	-0.096	-0.004	-0.020	-0.093	-0.118	-0.005	-0.361

Note: The 'shock' and 'GDP change' values are in %.

Source: author's representation

The differentiated tax impact on individual sectors proves the results from the previous section. The homogenous taxation slows down the GDP growth significantly more to the heterogeneous taxation. The economic growth drops by 0.361 percent if the uniform tax rate grows by 10 percent while by less than 0.1 percent if the rates respect the differentiated industrial susceptibility. Next, the

outcome in Table 6 suggests the lowest taxation for the Other, Industrial, and Service sectors. Table 4 supports these findings by showing the highest economic share of the intermediate inputs from these industries; therefore, taxation of these critical inputs could trigger a potential sizable network effect, as shown in Figure 7.

These results from Table 6 indicate significant negative consequences for the Czech GDP from the EU harmonization strategy abolishing the reduced rates. For comparison, the 10 percent increase analysed above is equivalent to 2.1 pp. growth in the tax rate, while the reduced rate abolition would mean 11 and 6 pp. the tax rate increase for various commodities, books, groceries, medicine, and different so-called do-it-yourself (DIY) services such as babysitting, household cleaning, but also hairdressing. The European Commission study (Directorate General Taxation and Customs Union, European Commission, 2008) discusses positive lower taxation effects for the DIY services on productivity. The taxation drop motivates higher-skilled individuals to purchase these services to acquire more time for their work in more productive areas (Directorate General Taxation and Customs Union, European Commission, 2008). In sum, a lower rate for these DIY services helps the economy: first, indirectly, by supporting the specialization for the high and low-skilled labour, and secondly, directly, by reducing the VAT tax impact on the GDP by reducing taxation for the third sector that otherwise slows down the GDP growth the most in Figure 7. Table 6 also supports the Service sector's significant impact on the GDP growth and suggests a small tax rate for this sector. Next, the model results in Table 6 recommend the tax rate below average for the Agricultural products. The reduced tax rate for groceries would benefit disadvantaged socio-economic groups (low-income families, single-parent households).

Overall, the presented analysis favours the current Czech pattern with heterogeneous tax rates. The efficient tax system should respect the industrial production sensitivity to taxation in order to minimize distortions in resource allocation and consumption sensitivity for different goods to reduce distortions in households' spending patterns. The optimal policy mixes these findings with political intentions, such as helping low-income classes, support local production, and trade DIY activities. Finally, the simulation results also motivate policymakers to consider manufacturing goods and other services as candidates for lower tax rates.

Conclusions

This study provides a case study for the Czech economy which compares the European Union trend of the VAT tax rate harmonization with the current Czech tax-policy trend of tax rate differentiation. The recent pandemic has further intensified Czech ambitions to reduce the VAT taxation for various goods and services. Yet, policymakers lack a tool to evaluate the economic consequences of such measures for the affected local businesses. The previous research concentrates on the aggregate tax multiplier for the Czech economy but misses the tax analysis at

the industrial level. This study presents a model that allows the tax analysis for selected industries (sectors). The simulation outcome discloses the sectors which are most susceptible to the taxation changes and suggests the most beneficial tax differentiation scheme that would boom economic production. Therefore, the findings can help policymakers in questioning the tax harmonization strategy and in evaluating the tax heterogeneity to ease the pandemic effects. The research method concerns the input-output computable general equilibrium (CGE) model. The model accounts for eight industries: Agricultural sector, Construction, Industrial sector, Mining, Finance, Services, Energy sector, and other sector composed of all the remaining sectors.

The study results for uniform taxation prove similar to previous research. The VAT rate 10 percent increase results in 0.34 gross domestic product slowdown. However, the same VAT rate proves a differentiated effect on individual industries. First, the taxed industries contract their intermediate input demand from other sectors differently; where the most sensitive demand is for those intermediate inputs which are the least necessary for production. The demand for the Agricultural and the Mining intermediate inputs exhibits such high price elasticity and decreases the most in the presence of the VAT shock. Secondly, the VAT tax shock also affects consumers' spending structure differently across sectors. The low price-elasticity for demand of necessary goods, food, and electricity makes the Agricultural and the Energy sectors less susceptible to taxation. In contrast, purchases of goods from the Service sector decrease the most. Third, the production shrinkage also varies for individual industries when subjected to the VAT tax, with the Mining, the Industrial, and the Service being the most susceptible ones. Fourth, the effect of taxing different sectors on final GDP depends on the industry susceptibility, the industry share in the economy, and the tax pass-through to other industries. Other, Industrial and Service sectors represent the three most affected industries.

Finally, the model simulation suggests a tax rate distribution that minimizes the taxation impact on the GDP. Such tax rate re-distribution would mean the lowest tax rates for the same sectors that yield the highest GDP impact from the taxation; the Other, Industrial, and Service sectors. This scenario would lead to a significantly lower GDP drop relative to the uniform taxation situation. The Czech legislative trend to include several services into the reduced rate group conforms to these recommendations. The Industrial sector is another sector that policymakers should consider when trying to minimize the VAT distortionary effects. In sum, the findings favour the current Czech heterogeneous tax rates and support extending the lower tax group for other services and manufacturing products.

The subsequent research could provide a more disaggregated model accounting for more industries. Next, the presented model could be applied to measure the Czech income tax system and its effects on the labour market.

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Appendix

The model reduced form parameters derived from the steady-state equal:

$$\alpha_i = \frac{\bar{X}_i^p}{\sum_{j=1}^N \bar{X}_j^p}$$

$$\beta_{l,j} = \frac{\bar{F}_{l,j}}{\sum_{k=1}^M \bar{F}_{k,j}}, \text{ with l-th and k-th production factors}$$

$$b_j = \frac{\bar{Y}_j}{\prod_{l=1}^M \bar{F}_{l,j}^{\beta_{l,j}}}$$

$$ax_{i,j} = \frac{\bar{X}_{i,j}}{\bar{Z}_j}$$

$$ay_j = \frac{\bar{Y}_j}{\bar{Z}_j}$$

$$\mu_i = \frac{G_i}{\sum_{j=1}^N \bar{G}_j}$$

$$\lambda_i = \frac{\bar{I}_i}{\bar{S}^p + \bar{S}^g + \bar{S}^f}$$

$$\delta m_i = \frac{(1 + \tau_i^m)(\bar{M}_i)^{1-\eta_i}}{\left((1 + \tau_i^m)\bar{M}_i^{1-\eta_i} + \bar{D}_i^{1-\eta_i} \right)}$$

$$\delta d_i = \frac{\bar{D}_i^{1-\eta_i}}{\left((1 + \tau_i^m)\bar{M}_i^{1-\eta_i} + \bar{D}_i^{1-\eta_i} \right)}$$

$$\gamma_i = \frac{\bar{Q}_i}{\left(\delta m_i \bar{M}_i^{\eta_i} + \delta d_i \bar{D}_i^{\eta_i} \right)^{\frac{1}{\eta_i}}}$$

$$\xi x_i = \frac{\bar{E}_i^{1-\phi_i}}{\left(\bar{E}_i^{1-\phi_i} + \bar{D}_i^{1-\phi_i} \right)}$$

$$\xi d_i = \frac{\bar{D}_i^{1-\phi_i}}{\left(\bar{E}_i^{1-\phi_i} + \bar{D}_i^{1-\phi_i} \right)}$$

$$\theta_i = \frac{\bar{Z}_i}{\left(\xi x_i \bar{E}_i^{\phi_i} + \xi d_i \bar{D}_i^{\phi_i}\right)^{\frac{1}{\phi_i}}}$$

$$\sigma_p = \frac{\bar{S}^p}{\frac{\sum_{l=1}^M \bar{F}_l}{\bar{S}^g}}$$

$$\sigma_g = \frac{\bar{S}^p}{\left(\bar{T}^w + \sum_{j=1}^G \bar{T}_j^y + \sum_{j=1}^G \bar{T}_i^m\right)}$$