Optimal taxation and monitoring in an economy with matching frictions and underground activities

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Abstract

This short paper shows the interdependence of taxation and monitoring policy in a search and matching model of equilibrium unemployment with an underground sector. More precisely, from a social welfare standpoint, two options are available to the policy maker: she/he may either substitute a tighter monitoring with a higher penalty or enforce both a higher taxation and an increased monitoring.

Key words: optimal taxation, tax evasion, underground economy, job search theory

JEL Classification: E26, H21, H26, J64

1. The interdependence of taxation and monitoring in a matching framework

The study of the effects of economic policies such as taxation, monitoring and punishment on the size of the underground economy and the level of involuntary unemployment is the focus of matching-type models with an underground sector[1]. However, a great many of these models do not discuss how the policy parameters can be optimally set by a policy maker who wants to maximize social output (shadow employment produces positive added value so some tolerance should be expected from an efficiency standpoint). As an exception, Boeri and Garibaldi (2002) show that the optimal taxation problem does not affect the policy-maker's choice with respect to the optimal monitoring of underground activities, whereas optimal taxation is affected by the optimal choice of monitoring (the *separability result*). Instead, this paper shows that the

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[1] The extension of matching models of unemployment to the underground economy is straightforward since unemployment and underground employment are strongly connected and interdependent phenomena (Lisi, 2010). For an overview of these models see Lisi (2011).

optimal monitoring problem is also affected by the optimal choice of taxation. Precisely, there are two options available to the policy maker: s/he may either substitute a tighter monitoring with a higher penalty or enforce both a higher taxation and an increased monitoring.

To make our point as simply as possible, we consider a basic matching framework *à la* Pissarides (2000) with a continuum of homogeneous workers of measure one. The creation of employment occurs in a labour market with matching frictions. As usual (see Pissarides, 2000; Petrongolo and Pissarides, 2001), an aggregate matching function is used to summarize these frictions. Precisely, the number of job matches formed per unit of time is $m = m(u, v)$, where u is the number of unemployed workers and v is the number of vacancies. The matching function is strictly increasing but concave in both arguments and displays constant returns to scale. It follows that the labour market tightness is given by $\theta_i = v_i / u$, where the subscript $i \in \{f, s\}$, with *f* = *formal* and *s* = *shadow*, denotes the type of firm (see below). Hence, $q(\theta_i) \equiv m\{v_i, u\}/v_i = m\{1, \theta_i^{-1}\}$ and $g(\theta_i) \equiv m\{v_i, u\}/u = m\{\theta_i, 1\}$, with $i \in \{f, s\}$, are the probability of filling a vacancy and of finding a job, respectively.[2]

We consider two types of firms, thus forming two sectors. Formal firms (with $i = f$) have to pay taxes τ, whereas shadow firms (with $i = s$) enjoy tax evasion.

To ensure that unemployment exists in steady state, it is assumed that job destruction occurs at the exogenous rate δ. Furthermore, since the underground activities are detected and repressed by the government at the exogenous rate ρ, the overall job destruction rate in the shadow sector is higher, i.e. $(\delta + \rho)$. Therefore, in steady state these matching and job destruction rates allow us to describe the evolution of employment in the course of time ($\dot{n}$):

$$\dot{n}_f = g(\theta_f) \cdot (1 - n_f - n_s) - \delta \cdot n_f \tag{1}$$

$$\dot{n}_s = g(\theta_s) \cdot (1 - n_f - n_s) - (\delta + \rho) \cdot n_s \tag{2}$$

where n_f and n_s are the steady state employment rates, and $1 - n_f - n_s = u$ is the unemployment identity.

[2] Standard technical assumptions are assumed, i.e. $\lim_{\theta \to 0} q(\theta) = \lim_{\theta \to \infty} g(\theta) = \infty$, and $\lim_{\theta \to 0} g(\theta) = \lim_{\theta \to \infty} q(\theta) = 0$.

The Bellman equations specified to find infinite horizon steady-state solutions are: [3]

Value of …	***Underground sector***	***Regular sector***
a vacancy	$rV_s = -c_s + q(\theta_s)\cdot(J_s - V_s)$	$rV_f = -c_f + q(\theta_f)\cdot(J_f - V_f)$
a filled job	$rJ_s = y_s - w_s + (\delta + \rho)\cdot(V_s - J_s)$	$rJ_f = y_f - w_f - \tau + \delta\cdot(V_f - J_f)$
searching for a job	$rU_s = b + g(\theta_s)\cdot(W_s - U_s)$	$rU_f = b + g(\theta_f)\cdot(W_f - U_f)$
being employed	$rW_s = w_s + (\delta + \rho)\cdot(U_s - W_s)$	$rW_f = w_f + \delta\cdot(U_f - W_f)$

where r is the exogenous discounted rate; c_i is the vacant job cost; y_i is the match productivity, w_i is the wage rate and b is the benefit of being unemployed.

As usual (see Pissarides, 2000), the equilibrium value of labour market tightness is given by the *free-entry condition* or *zero profit condition* for firms (i.e. $V_i = 0$):

$$\frac{y_f - w_f - \tau}{r + \delta} = \frac{c_f}{q(\theta_f)} \tag{3}$$

$$\frac{y_s - w_s}{r + \delta + \rho} = \frac{c_s}{q(\theta_s)} \tag{4}$$

with $\partial\theta_f / \partial\tau < 0$, $\partial\theta_s / \partial\rho < 0$, $\partial^2\theta_f / \partial\tau^2 > 0$, $\partial^2\theta_s / \partial\rho^2 > 0$ given the properties of matching function.[4]

By definition, an equilibrium with matching frictions requires positive and finite values of tightness ($0 < \theta_i < \infty, \forall\, i$). Whereas, in a competitive equilibrium the firm has no vacant job cost ($c_i = 0$), then the marginal condition for labour demand (i.e. the *free-entry* condition) reduces to the standard marginal productivity condition for employment in the steady state (Pissarides, 2000). Therefore, the competitive equilibrium outcome is $y_f - \tau = w_f$, in the formal sector, and $y_s = w_s$ in the shadow sector. It follows that as long as $y_f - \tau > y_s$ the underground sector is less attractive, but if $y_f - \tau < y_s$ then job matches

[3] The unemployed cannot search for jobs in both sectors at the same time (i.e. there is a directed search). However, irrespective of the sector, if an unemployed person fails to find a job, she/he falls back into the same unemployment pool.

[4] In case where taxes are levied proportionally as percentage of output the qualitative results of the analysis would not change since the productivity is the same for all firms in the formal sector.

can be induced to remain in the formal sector by an appropriate choice of taxation, since there is no role for monitoring rate (in fact, the competitive equilibrium outcome implies that $J_f = J_s = 0$).

As usual in a matching framework, wages are the outcome of a bilateral matching problem described by the *Nash bargaining solution*,

$w_i = arg\,max\{(W_i - U_i)^{\beta_i} \cdot (J_i - V_i)^{1-\beta_i}\} \Rightarrow (W_i - U_i) = \frac{\beta_i}{(1-\beta_i)} \cdot (J_i - V_i)$, with $i \in \{f, s\}$

where $\beta_i \in (0,1)$ is the bargaining power of workers. Simple manipulations thus yield the formulae of wages:

$$w_f = (1-\beta_f) \cdot rU_f(\theta_f) + \beta_f \cdot (y_f - \tau - rV_f(\theta_f)) \quad (5)$$

$$w_s = (1-\beta_s) \cdot rU_s(\theta_s) + \beta_s \cdot (y_s - rV_s(\theta_s)) \quad (6)$$

with $w_i'(\theta_i) > 0 \ \forall i$, since $V_i'(\theta_i) < 0$, and $U_i'(\theta_i) > 0 \ \forall i$.

Therefore, for given policy parameters τ and ρ, equations (1) – (6) together with the unemployment identity define a steady state (decentralised) equilibrium (with $\dot{n}_f = \dot{n}_s = 0$).

Now we derive the optimal level of taxation τ and monitoring ρ in a context in which a policy-maker maximizes the social output, where the latter also includes the output and the vacancy cost in the shadow sector. Furthermore, we consider the case in which taxation is used by the benevolent social planner to finance the benefit of being unemployed, i.e. $b \cdot u = \tau$. Following the textbook of Pissarides (2000), the social welfare function for an infinitely-lived economy is equal to the net output per job, minus vacancies costs, plus the benefit of being unemployed. The policy-maker thus maximizes the following programme: [5]

$$\Omega = \int_0^\infty e^{-rt}\left[(y_f - \tau) \cdot n_f + y_s \cdot n_s - c_f \cdot \theta_f \cdot u - c_s \cdot \theta_s \cdot u + \tau\right] dt$$

subject to: $\begin{cases} \dot{n}_f = g(\theta_f) \cdot u - \delta \cdot n_f \\ \dot{n}_s = g(\theta_s) \cdot u - (\delta + \rho) \cdot n_s \\ u = 1 - n_f - n_s \end{cases}$

[5] The social planner is not interested in wages, since wages only determine the output's distribution and distributional considerations are excluded from the social welfare function. Furthermore, the social planner is subject to the same matching constraints as firms and workers. Hence, the evolution of employment constrains social choices as well as private ones.

Let λ and μ be *co-state variables* (i.e. the *shadow values*) at the time t, so that the *Hamiltonian* (*H*) is:

$$H = \Big\{ e^{-rt} \cdot \left[\left(y_f - \tau \right) \cdot n_f + y_s \cdot n_s - c_f \cdot \theta_f \cdot \left(1 - n_f - n_s \right) - c_s \cdot \theta_s \cdot \left(1 - n_f - n_s \right) + \tau \right] + \ldots$$

$$\ldots + \lambda \cdot \left[g\left(\theta_f\right) \cdot \left(1 - n_f - n_s\right) - \delta \cdot n_f \right] + \mu \cdot \left[g\left(\theta_s\right) \cdot \left(1 - n_f - n_s\right) - \left(\delta + \rho\right) \cdot n_s \right] \Big\}$$

The solution to this dynamic maximization problem requires that: [6]

$$\frac{\partial H}{\partial \tau} \Rightarrow -n_f - c_f \cdot \frac{\partial \theta_f}{\partial \tau} \cdot u + 1 + \lambda \cdot \frac{\partial g\left(\theta_f\right)}{\partial \tau} \cdot u = 0 \qquad \text{(I)}$$

$$\frac{\partial H}{\partial \rho} \Rightarrow -c_s \cdot \frac{\partial \theta_s}{\partial \rho} \cdot u + \mu \cdot \left[\frac{\partial g\left(\theta_s\right)}{\partial \rho} \cdot u - n_s \right] = 0 \qquad \text{(II)}$$

$$\frac{\partial H}{\partial n_f} \Rightarrow \left(y_f - \tau\right) + c_f \cdot \theta_f + c_s \cdot \theta_s - \lambda \cdot \left[g\left(\theta_f\right) + \delta \right] - \mu \cdot g\left(\theta_s\right) = -\left(\dot{\lambda} - r \cdot \lambda\right) \qquad \text{(III)}$$

$$\frac{\partial H}{\partial n_s} \Rightarrow y_s + c_f \cdot \theta_f + c_s \cdot \theta_s - \lambda \cdot g\left(\theta_f\right) - \mu \cdot \left[g\left(\theta_s\right) + \left(\delta + \rho\right) \right] = -\left(\dot{\mu} - r \cdot \mu\right) \qquad \text{(IV)}$$

From the previous conditions, the result of interdependence between the optimal choice of taxation and the optimal choice of monitoring is straightforward, since conditions (I) – (IV) set up a set of four equations of four unknowns (τ, ρ, λ, μ). Furthermore, the sign of the relationship between taxation and monitoring crucially depends on λ and μ.

In steady state (with $\dot{\lambda} = \dot{\mu} = 0$), by using (III) and (IV) algebraic manipulations give:

$$\lambda = \lambda(\tau, \rho) = \frac{g\left(\theta_s\right)\left(y_f - \tau - y_s\right) + \left(r + \delta + \rho\right) \cdot \left(y_f - \tau + \theta_s \cdot c_s + \theta_f \cdot c_f\right)}{\left(r + \delta\right) \cdot \left(\rho + g\left(\theta_s\right) + r + \delta\right) + g\left(\theta_f\right) \cdot \left(r + \delta + \rho\right)} \qquad \text{(III b)}$$

$$\mu = \mu(\tau, \rho) = \frac{g\left(\theta_f\right)\left(\tau - y_f + y_s\right) + \left(r + \delta\right) \cdot \left(y_s + \theta_s \cdot c_s + \theta_f \cdot c_f\right)}{\left(r + \delta\right) \cdot \left(\rho + g\left(\theta_s\right) + r + \delta\right) + g\left(\theta_f\right) \cdot \left(r + \delta + \rho\right)} \qquad \text{(IV b)}$$

[6] Besides the *transversality conditions*, i.e. $\lim_{t \to \infty} \lambda \cdot e^{-rt} \cdot n_f = 0$ and $\lim_{t \to \infty} \mu \cdot e^{-rt} \cdot n_s = 0$.

with:

$$\frac{\partial\mu}{\partial\tau}=\frac{\left(g(\theta_f)+(r+\delta)\cdot c_f\cdot\frac{\partial\theta_f}{\partial\tau}+\frac{\partial g(\theta_f)}{\partial\tau}\cdot(\tau-y_f+y_s)\right)}{g(\theta_f)\cdot(r+\delta+\rho)+(r+\delta)\cdot(r+\delta+\rho+g(\theta_s))}$$

$$-\frac{(r+\delta+\rho)\cdot\frac{\partial g(\theta_f)}{\partial\tau}\cdot\left((r+\delta)\cdot(y_s+c_s\cdot\theta_s+c_f\cdot\theta_f)-g(\theta_f)(y_f-\tau-y_s)\right)}{\left(g(\theta_f)\cdot(r+\delta+\rho)+(r+\delta)\cdot(r+\delta+\rho+g(\theta_s))\right)^2}>0$$

under the following two conditions: (i) $y_f-\tau-y_s=0$,[7] which is no better than an equilibrium condition, namely, in equilibrium an entrepreneur operates indifferently in one of the two sectors if $y_f-\tau=y_s$; (ii) $(r+\delta)$ is sufficiently small, which is a realistic condition since small calibration values are usual in the literature. More precisely, $(r+\delta)$ ranges between 0.112 (Shimer, 2005) and 0.18 (Boeri and Garibaldi, 2007).

Nevertheless, the signs of $\frac{\partial\mu}{\partial\rho}$, $\frac{\partial\lambda}{\partial\tau}$, and $\frac{\partial\lambda}{\partial\rho}$ remain indeterminate. Therefore, we exploit the limited available information. Precisely, total differentiation of condition (II) gives:

$$\frac{d}{d\tau}\left(-c_s\cdot\frac{\partial\theta_s}{\partial\rho}\cdot u+\mu(\tau,\rho)\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]\right)=\frac{\partial\mu(\tau,\rho)}{\partial\tau}\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]<0$$

and,

$$\frac{d}{d\rho}\left(-c_s\cdot\frac{\partial\theta_s}{\partial\rho}\cdot u+\mu(\tau,\rho)\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]\right)=$$

$$-c_s\cdot\frac{\partial^2\theta_s}{\partial\rho\,\partial\rho}\cdot u+\frac{\partial\mu(\tau,\rho)}{\partial\rho}\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]+\mu(\tau,\rho)\cdot\left[\frac{\partial^2 g(\theta_s)}{\partial\rho\,\partial\rho}\cdot u-\underbrace{\frac{\partial n_s}{\partial\rho}}_{<0}\right]$$

[7] In an equilibrium with matching frictions wages adjust in order to reach an equilibrium without corner solutions, since $\partial w_i/\partial\theta_i>0$ (*wage setting*) but $\partial\theta_i/\partial w_i<0$ (*free-entry condition*) $\forall\, i$. In fact, if $J_f>J_s$, since $\rho>0$, then the resultant growth in formal vacancies increases formal wages ($\partial w_i/\partial\theta_i>0$) but in turn this effect reduces tightness in formal sector ($\partial\theta_i/\partial w_i<0$). Therefore, the two effects offset each other.

which is $\begin{cases} >0 & \text{if } \dfrac{\partial \mu}{\partial \rho} < 0 \\ <0 & \text{if } \dfrac{\partial \mu}{\partial \rho} > 0 \text{ and } \mu \approx 0 \end{cases}$

since c_s is sufficiently low or even zero.[8]

Therefore, we may obtain both negative and positive relationship between taxation and monitoring policy. As a result, from the social welfare standpoint, two options are available to the policy maker: she/he may either substitute a tighter monitoring with a higher fine (the penalty is usually assessed on the level of taxes) or enforce both a higher taxation and an increased monitoring (this could be recommended in countries like Italy where public spending is very inelastic, public debt is very large and tax morale is very low).

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[8] The vacancy job cost in the underground sector should be very low or even zero, since ease of entry is one of the criteria used to define the informal sector (Gërxhani, 2004).