Day-of-the-week and month-of-the-year effects on French Small-Cap Volatility: the role of asymmetry and long memory

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Abstract

Small-cap stocks are characterized by high volatility and offer investors the opportunity to earn higher returns. This paper empirically investigates the impact of the day-of-the-week and the month-of-the year effects on the volatility of daily and monthly CAC SMALL returns in Paris stock market during the period from 1999 to 2015. We propose the SEMIFARMA-SD-GJR-GARCH model, which incorporates stochastic trend, deterministic nonparametric trend, short-range, long-range dependence and seasonal dummy asymmetric GARCH errors. The main findings of this study are that the coefficients of the SEMIFARMA-SD-GJR-GARCH model including the long memory coefficient in the mean equation and the seasonal asymmetry in the variance equation are highly significant and the GJR-GARCH model without seasonal dummies is dominated by the GJR-GARCH model with seasonal dummies (SD-GJR-GARCH). The results indicate that the day-of-the-week and the month-of-the-year effects detected on volatility seem to improve the volatility forecasts. These results support the arbitrage opportunity hypothesis for realizing abnormal returns, and support the inefficiency of CAC small capital market.

Keywords: SEMIFARMA model, SD-GJR-GARCH model, seasonal anomalies, asymmetric volatility, small capitalization

Introduction

The listed stocks of small-cap have received considerable attention in research centres for several years due to their special characteristics of the impact of size, which is related to information efficiency and the cost of equity capital or expected

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return. Investor's professional is aware that small-cap stocks offer characteristics that call into question the use of theories and pricing models of capital assets. Speculators, arbitrators and hedgers seek to increase their returns and/or reduce their portfolio risks in the stock markets. For making investment decisions, they estimate the expected return and risk by using technical or fundamental analysis. According to the efficient market hypothesis, there is no significant difference between the estimated value and the intrinsic value of the financial assets. In practice, there have been many anomalies in stock returns which the existing theories have not been able to explain. The capital market efficiency theory assumes that asset prices always reflect all relevant available information (Fama, 1970). In practice, many studies have proved that the anomalies in the share prices generate seasonal patterns (Wachtel, 1942; Cross, 1973; French, 1980; Gibbons and Hess, 1981; Rogalski, 1984; Smirlock and Starks, 1986; Thaler, 1987; Schwert, 2003; Akbalik and Ozkan, 2017; Zhang *et al*., 2017) and proved the existence of Monday and Friday effect or January effect (Ariel, 1987). The presence of patterns in stock returns indicates an important observation that the capital market is inefficient and produces almost arbitrage opportunity. Therefore, previous studies provide sufficient evidence of the impact of the day and month, on developed markets, developing, commodity markets, precious metals markets and even on the interbank market. In other words, the existence of any seasonal anomaly in volatility will have important effects on investment strategies. So, prudent investors can capitalize on the anomaly to make abnormal volatility. The presence of calendar effect anomalies is an evidence of stock market inefficiency and will violate the efficient market hypothesis.

The share returns are determined by many factors, such as volume, sales growth and liquidity but the day of the week and the month of the year may have negative or positive effects on the return and its volatility. The theme of seasonal effects is one of the most influential issues in the stock market analysis and its anomalies. A significant impact for the day or the month will be very beneficial for traders to take advantage of speculation, arbitrage or hedging opportunities.

The stock returns anomalies: day-of-the-week and month-of-the-year effects have been tested by many literatures at the level of a single country (Pena, 1995; Liu and Li, 2010; Abdalla, 2012) or of several countries (Basher and Sadorsky, 2006; Anwar and Mulyadi, 2012; Dicle and Levendis, 2014; Kostyantyn *et al.,* 2019). The authors highlight this subject at the sector level (Bampinas, Fountas and Panagiotidis (2016), or at the level of individual stocks (Liu and Li, 2010). In the beginning, the research focused on the stock markets of developed countries (Cross, 1973; French, 1980; Berument and Kiymaz, 2001) and later on in developing countries (Aggarwal and Rivoli, 1989; Al-Loughani and Chappell, 2001; Gbeda and Peprah, 2017). At the level of 51 equity markets in 33 countries over January, 2000 to December, 2007, Dicle and Levendis (2014) test empirically the DOW effects for individual stocks in the international equity markets using the GARCH (1,1) and VAR models. Bampina *et al*., (2016) use the rolling regression techniques for testing the daily seasonality in the European real estate sector in 12 countries during the period 1990-2010 using the symmetric and asymmetric GARCH models in the mean equation that provide evidence in favour of the day-of-the-week effect. Abdalla (2012) uses the OLS and the GARCH models in the Khartoum stock exchange index over the period of the 2nd January 2006 to the 30th October 2011, including the week effect in the conditional mean and volatility equations. Basher and Sadorsky (2006) analyse the day-of-the-week effect in 21 emerging stock markets namely Argentina, Brazil, Chile, Colombia, India, Indonesia, Israel, Jordan, Korea, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Sri Lanka, Taiwan, Thailand, Turkey, Venezuela and South Africa. They show that while the day-of-the-week effect is absent in the majority of emerging stock markets under study, some emerging stock markets exhibit strong day-of-the-week effects even after accounting for conditional market risk. Anwar and Mulyadi (2012) use the EGARCH models to analyse the day-of-the-week effects in Indonesia, Singapore, and Malaysia stock markets in order to identify the existence of anomalies in the three countries over the period from Jul 1st, 2003 to Jun 30th, 2008. In Australian stock markets, Liu and Li (2010) study the day-of-the-week effects in the top 50 Australian companies across different industry sectors during the period of January 2001 through June 2010. This study finds that weekday anomalies are mixed across companies and industries. This result lends some support to the view of reversing weekend effects. Akbalik and Ozkan (2017) perform the Kruskal-Wallis and Wilcoxon rank tests to examine the presence of day of the week effect in the stock markets of Brazil, India, Indonesia, Turkey and South Africa during the period from the 2nd of January 2009 to the 31st of December 2015. Yang and Chen (2014) take into account the long memory and the impact of day-of-the-week for modelling the conditional volatility with the high-frequency data using the HAR-D-FIGARCH. Recent studies (Akbalik and Ozkan, 2017; Boubaker *et al*., 2017; Srinivasan, 2017) show that these anomalies are absent in developed markets and are important in emerging markets. But there are no studies that tested these anomalies in the small capitalization, according to our knowledge. On the other hand, most studies use the generalized autoregressive conditional heteroscedasticity (GARCH) models for testing the DOW or the month-of the year effects on return or volatility time series Akgiray (1989); Baillie and DeGennaro (1990); Nelson (1991); Campbell and Hentschel (1992); Glosten Jagannathan and Runkle (1993); Anwar and Mulyadi (2012); Abdalla (2012); Yang and Chen (2014); Dicle and Levendis (2014); Boubaker *et al*. (2017) and Srinivasan (2017).

The dynamics of stock returns modelling has been examined by many authors. Some of them focus on the return by including the seasonal dummies while others introduce the seasonal component into the volatility. In this sense, some studies confirm the existence of the seasonality of financial assets return and volatility, which implies that they have a similar behaviour on some days or during some months. Kumar and Singh (2008) study the volatility, the risk premium and the seasonality in the risk-return relation of the Indian stock and commodity markets using GARCH-in-Mean model introduced by Engle *et al*. (1987). McGowan and Ibrihim (2009) evaluate the weak form efficiency of the Russian stock market by testing for a day-of-the-week effect using GARCH modelling by introducing the seasonal component into the conditional mean equation. Charles (2013) investigates the impact of the day-of-the-week effect in major international stock markets empirically using GARCH family models in a forecast framework. Terraza (2010) studies non-linear dynamics in the CAC 40 stock index by combining seasonality, persistence and asymmetric effects to model the conditional volatility. Chirila and Chirila (2015) test the relationship between return and asymmetric seasonal volatility.

The limitation of these works is that they ignore the existence of long memory structure in the conditional mean. The long range dependence phenomenon has raised a difficult problem in the financial time series analysis. The presence of long memory components in the financial data is a key issue which has important implications for risk management and portfolio allocation. More precisely, many studies find that the short-memory time series are contaminated by deterministic trend or level shifts, which display many of the same properties of long-memory time series (Boubaker and Sghaier, 2014). Beran and Feng (2002, a; b) add a nonparametric deterministic trend into the ARFIMA model and thus develop the SEMIFARMA (Semiparametric fractional autoregressive Moving average) model. Then, Feng *et al*. (2007) propose the SEMIFARMA-GARCH model to allow for conditional variance. One of the most important features of financial assets is the presence of seasonality and asymmetry in the volatility. The added flexibility of the ARFIMA model is the use of a separate parameter to capture long-run dependence. The ARFIMA parameters can capture both low-frequency and high-frequency components in the spectral density. Finally, the simplicity and flexibility of the GARCH may outweigh the gain in forecasting the performance of the ARFIMA model. More importantly, the joint estimations of an ARFIMA model in the mean equation and GARCH-type models in the variance equations reveal that cyclical components appear to be well described by seasonal asymmetric models. So, we contribute to the literature by proposing a model that combines the SEMIFARMA part with seasonal asymmetric volatility component to examine the stock market returns. In other words, we take into account the semiparametric fractional ARIMA in the conditional mean and the seasonality in the asymmetric conditional volatility.

This literature concerned developed and developing countries, sectors, individual stocks, commodity market and derivatives. In our knowledge, the small capitalization is not present in this subject. There is not published research in the financial literature investigating the presence of long memory in mean and these seasonal anomalies in volatility by employing small capitalization data.

This paper focuses on two types of anomalies in the stock markets, the first on the size effect as it highlights the small-cap, and the second on the Day-of-the-week and month-of-the-year effects on conditional volatility. In addition, this study builds the econometric model for forecasting abnormal returns with long memory in the return series by using the SEMIFARMA-SD-GJR-GARCH models.

The main objective of this paper is to investigate the existence of long memory in the stock returns and the day-of-the-week and the Month-of-the year effects in the French stock market volatility using daily and monthly observations of CAC SMALL price index series from the Paris Stock Exchange. This work aims to theoretically inspect the meaning, the boundaries, the long memory and seasonal anomalies phenomenon that affect the French Stock Exchange Market. After that, we analytically evaluate long memory and the seasonal anomalies phenomenon and its possible fitting with the SEMIFARMA-SD-GJR-GARCH model. The conditional variance process may depend on day-of-week levels and yearly and half-yearly cosine waves with deviations from these deterministic functions modelled by a GARCH process with GED.

The remainder of this paper is organized as follows: Section 2 describes the SEMIFARMA-SD-GJR-GARCH model used throughout our study, followed by the presentation of the data used and our estimation results shown in Section 3. In section 4, we evaluate the volatility forecasting performance of the best fitting GARCH Models in the French stock market, including the semiparametric long memory in the mean equation. Finally, section 5 concludes the paper.

2. Presentation of the SEMIFARMA-SD-GJR-GARCH model

Although volatility has been considered in most empirical studies on financial markets, the seasonal factors and the leverage effects in the variance equation also seem to be important. The evidence indicates that there is still value in these anomalies, such as the January effect, monthly phenomenon, turn-of-the-month and first-half-of-the-month, turn-of-the-year, holiday and golden week effects still exist in the turbulent markets of the early part of the 21st century. The investigated evidence on several seasonal regularities on the French Stock Exchange using data is measured by the S&P 500 Index. The results expected are useful for investors who wish to tilt portfolios and for speculators who wish to trade the effects. On the other hand, many authors propose to determine whether incorporating asymmetric effects of positive and negative shocks on volatility adds a new twist to the existing understanding of the day-of-the-week and the month-of-the-year effects on volatility. Focusing on the impacts of positive and negative shocks, the GJR-GARCH model possesses an asymmetric news impact curve and uses a threshold function to capture volatility asymmetry.

In the SEMIFARMA-GJR-GARCH models, see Beran and Feng (2002a) for the SEMIFAR part and Glosten, Jagannathan and Runkle (1993) for the GJR-GARCH part, we take into account the seasonal behaviour of the financial series by including seasonal dummy variables in the asymmetric conditional volatility equation. $\{Y_t\}$ is a fractional semiparametric process with Seasonal dummy GJR-

GARCH error, called SEMIFARMA-SD-GJR-GARCH if it verifies the following relationship:

$$\varphi(B)(1-B)^{d_1}\left\{(1-B)^d Y_t - g(t)\right\} = c + \theta(B)\varepsilon_t \quad (1)$$

and $$\varepsilon_t = u_t\sigma_t \ , \quad \sigma_t > 0 \ , \ u_t \sim iid(0,1) \quad (2)$$

with $$\sigma_t^2 = \omega + \alpha(B)\varepsilon_t^2 + \beta(B)\sigma_t^2 + \sum_{k'=1}^{r'} \gamma_{k'} \varepsilon_{t-k'}^2 I_{\{\varepsilon_{t-k'}<0\}} + \sum_{s=1}^{m-1} \delta_j D_t^j \quad (3)$$

where $$\alpha(B) = \sum_{k=1}^{r} \alpha_k B^k \ , \ \beta(B) = \sum_{l=1}^{z} \beta_l B^l \quad (4)$$

and $\omega > 0, \ \alpha_k \geq 0, \ \beta_l \geq 0, \ \gamma_{k'} \geq 0, \ k = 1,...,r, \ l = 1,...,z \ , \ k' = 1,...,r'$

Where $D_t^1, D_t^2, ..., D_t^{m-1}$ are seasonal dummies and *B* is the lag operator, $(1-B)^{d_1} = \sum_{\tau=0}^{¥} \frac{\Gamma(k-d_1)B^\tau}{\Gamma(-d_1)\Gamma(k+1)}$ and $\Gamma(.)$ is the gamma function. Furthermore, the polynomials $\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - .... - \varphi_p B^p$, $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - .... - \theta_q B^q$, $\alpha(B) = \alpha_1 B + \alpha_2 B^2 + ... + \alpha_r B^r$ and $\beta(B) = \beta_1 B + \beta_2 B^2 + ... + \beta_z B^z$ have orders p, q, r and z respectively with all their roots outside the unit circle., d_1 is an integer: $d \in \{0,1\}$. t is a time trend and g : [0, 1] → R is the smoothing nonlinear deterministic trend function that can be estimated by the kernel methodology in the case of long memory errors (see Hall and Hart (1990), Ray and Tsay (1997) and Beran (1999):

$$(1-B)^d Y_t = g(t) + X_t \quad (5)$$

It is equivalent to $Y_t = g(t) + X_t$ if $d = 0$ and to $Y_t - Y_{t-1} = g(t) + X_t$ if $d = 1$, where X_t is a process of long memory stationary errors if $d_1 > 0$. We consider the polynomial kernel defined by (see Chikhi *et al*. (2013)):

$$K(x) = \sum_{l=0}^{\tau} \alpha_l x^{2l} \text{ with} |x| \leq 1 \quad (6)$$

and $K(x) = 0$ if $|x|$>1. Here we have $\tau \in \{0, 1, 2, \ldots\}$ and the α_l coefficients are such that $\int_{-1}^{1} K(x)dx = 1$ (see Beran and Feng, 2002a; b) for details on the estimation method).

The process is stationary and invertible, $-\frac{1}{2} < d_1 < \frac{1}{2}$, and u_t is an i.i.n. process. $I_{\{\varepsilon_{t-k'}<0\}}$ is an indicator variable taking the value one if the shocks are negative ($\varepsilon_{t-k'} < 0$) and the value zero if the shocks are positive ($\varepsilon_{t-k'} \geq 0$). It captures the asymmetric impacts by the sign of the indicator term to reflect a different influence between good news and bad news. The nature of shocks has therefore an effect on the conditional variance. For $\gamma_{k'} \neq 0$, the impact of news will be asymmetric (presence of level's effect), and if $\gamma_{k'} > 0$, the bad news will tend to accentuate the volatility. When $\gamma_{k'} = 0$, the SD-GJR-GARCH model corresponds to a seasonal dummy GARCH model (SDGARCH).

Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989), Bollerslev *et al*. (1992), Palm (1996), Pagan (1996), Palm and Vlaar (1997), Davidson (2004), Kwan, and Lux and Morales-Arias (2010) show that the distribution of the innovation ε_t is not Gaussian, and the commonly used Gaussian quasi-MLE is inefficient. Terasvirta (1996) also suggests that normal GARCH models cannot capture the full extent of excess kurtosis in high-frequency data. The Student's t distribution better captures the observed kurtosis in the stock returns series. Gao, Zhang and Zhang (2012) compare the kurtosis coefficients and the standard deviations between the autocorrelation given by their model and the real autocorrelation. They conclude that the GED-GARCH model is better than t-GARCH, and t-GARCH is better than Normal GARCH. In order to estimate the SEMIFARMA-SD-GJR-GARCH model, we use the maximum likelihood estimator based on GED distribution (See Nelson, 1991).

$\theta = (c, \omega, \alpha_1, .., \alpha_r, \delta_1, .., \delta_{m-1}, \varphi_1, .., \varphi_p, \theta_1, .., \theta_q, d_1, \beta_1, .., \beta_z, \gamma_1, .., \gamma_{r'})'$ is the parameter vector of models (1) and (3) and where θ is a suitable compact set in $R^{p+q+r+z+m+r'+2}$. Suppose the innovation ε_t follows Generalized Error Distribution (GED), which is a symmetric distribution that can be both leptokurtic and Platykurtic depending on the degree of freedom ν ($\nu > 1$). The density is given by:

$$f_\nu(x,\nu) = \frac{\nu e^{-\frac{1}{2}\left|\frac{x}{\lambda}\right|^{\nu}}}{\lambda 2^{(\nu+1)/\nu}\,\Gamma(1/\nu)} \tag{7}$$

where $\Gamma(.)$ is the gamma function and

$$\lambda = \left(\frac{2^{-2/\nu}\,\Gamma(1/\nu)}{\Gamma(3/\nu)}\right)^{1/2} \tag{8}$$

The density function of GED with unit variance is written as:

$$f_{\nu}(x,\nu) = \frac{\nu e^{-\frac{1}{2}\left|\frac{x}{\lambda}\right|^{\nu}}}{\lambda 2^{(\nu+1)/\nu}\,\Gamma(1/\nu)\sqrt{\nu/(\nu-2)}} \tag{9}$$

For $\nu = 2$, the GED is a standard normal distribution whereas the tails are thicker than in the normal case when $\nu < 2$, and thinner when $\nu > 2$.

The log likelihood function of $y_1, ..., y_n$ conditional on $y_0, y_{-1}, ...$ is given by:

$$L_n(\theta,\nu) = \sum_{t=1}^{n}\left[\log\left(\frac{\nu}{\lambda}\right) - \frac{1}{2}\left|\frac{e_t(\theta)}{\lambda\sqrt{\sigma_t^2(\theta)}}\right|^{\nu} - \left(1+\nu^{-1}\right)\log(2) - \log\Gamma\left(\frac{1}{\nu}\right) - \frac{1}{2}\log\sigma_t^2(\theta)\right] \tag{10}$$

The log-likelihood function is maximized with respect to the unknown parameter vector θ (the same procedure as in the Gaussian MLE case). The maximum likelihood estimator (MLE) can be defined as:

$$\hat{\tau}_n = \left(\hat{\theta}', \hat{\nu}\right)' = \underset{\theta\in\Theta,\nu\in V}{\arg\max}\, L_n\left(\theta,\nu\right) \tag{11}$$

where $V \subset (2,\infty)$ is a compact set and the true degrees of freedom ν_0 is an interior point of V.

3. Data characteristics and main results

The data considered in this paper consists of index data for the Paris stock market CAC SMALL (France). 17 years of daily and monthly index data was downloaded from Yahoo Finance covering a historical period from January, 1999 to December, 2015 (n=204 for monthly frequency and n = 3233 for daily frequency).

The stock prices have the potential information content, so we used daily and monthly data to compare the results in terms of the information content of stock prices. On the other hand, a long period from 1999 to 2015 was used for long memory tests and the volatility clustering. However, because of the availability of data, the small French capitalization market was chosen for the authors of a study on the French financial market.

In addition, 50% of the outflows from mid small cap equity funds in the euro zone were concentrated in France (source Exane). According to the classification of Small-cap, there are large companies belonging to all industrial sectors, there are many leading indicators of the Small-cap market, and the index contains a long list of companies. These indicators reflect the overall performance of the small-cap sector. The most important indicators are the Russell 2000 index, the MSCI USA Small Cap Index, or the S&P Small Cap 600 Index, the CAC Small-cap index. The latter is the focus of our attention in this study.

The performance of small-cap stocks in France, where the following figure outperforms its performance compared to medium and large capitalization in the last five years.

Figure 1. The performance of small-cap stocks, to medium and large capitalization in France

Source: Zone bourse (https://www.zonebourse.com, 25th SEP 2019)

Unit root tests results (Philips and Perron, 1988; Elliott, Rothenberg and Stock, 1996; Bierens, 1997; Breitung (2002) reported in table 1 show that the daily and monthly logarithmic series are characterized by the presence of unit roots. These two series are finally differenced to obtain the returns (see figure 1).

Table 1. Unit root tests

Frequency	Series	Philips-Perron	Elliott-Rothenberg-Stock	Bierens			Breitung
				β	A_m	F-test	
Daily	logarithmic	1.127 (-1.94)	0.018 (3.26)	-0.0019 [-1.759]	-6.864	1.139	0.01828 (0.0104)

	Returns	-51.023 (-1.94)	34.924 (3.26)	-0.8237 [-29.830]	-2463.286	296.611	0.00006 (0.0104)
Monthy	logarithmic	0.803 (-1.94)	0.270 (3.171)	-0.03618 [-2.278]	-10.735	1.761	0.04202 (0.0100)
	Returns	-10.333 (-1.94)	13.077 (3.171)	-0.69205 [-8.189]	137.657	22.356	0.0005 (0.01003)

Notes: H_0 : Unit root, (.): The asymptotic critical value at 5%. [.]: t-value. For Philips-Perron test, the optimal bandwidth: 6.08 for daily logarithmic series, 1.1 for daily returns series, 5.12 for monthly logarithmic series and 0.287 for monthly returns series. The spectral estimation method used is that of Andrews by Bartlett kernel. The table reports also the results of Bierens non-linear Augmented Dickey-Fuller and Breitung's nonparametric unit root tests. For Bierens unit root test, the unit root hypothesis is tested based on the t-statistic of β, the test statistic A_m and the F-test. The test statistic is the t-value of β. Breitung test only reports the critical values for n = 100, n = 250 and n = 500. Therefore, the critical values used here are the ones for n = 500. We accept the unit root hypothesis H_0 for daily and monthly logarithmic series and reject it for daily and monthly returns.
Source: authors calculations

The assumption of normality in returns is clearly rejected (see Table 2 and Figure 2). The two distributions are clearly leptokurtic and the observed asymmetry may indicate the presence of nonlinearities in the evolution process of daily and monthly returns. In addition, the ARCH-LM test result shows that CAC SMALL returns are characterized by the presence of an ARCH effect according to the results of normality test.

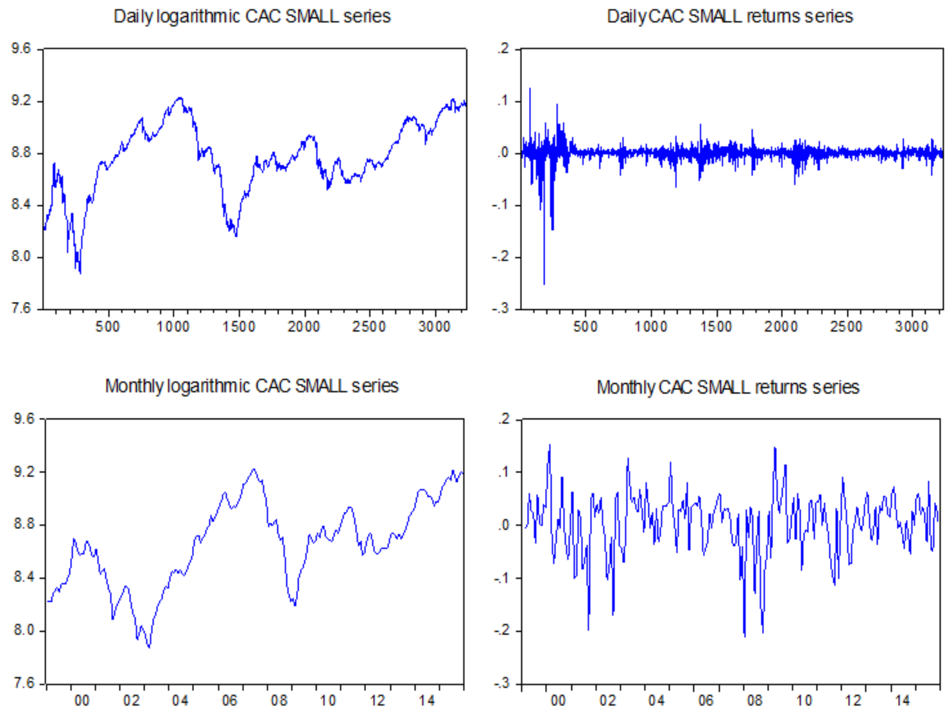
Table 2. Normality and ARCH-LM tests on returns

Frequency	Skewness	Kurtosis	Jarque-Bera statistic	ARCH(1)-LM
Daily returns	-4.036	86.544	948710.4	16.820
Monthly returns	-0.854	4.803	52.194	6.078

Source: authors calculations

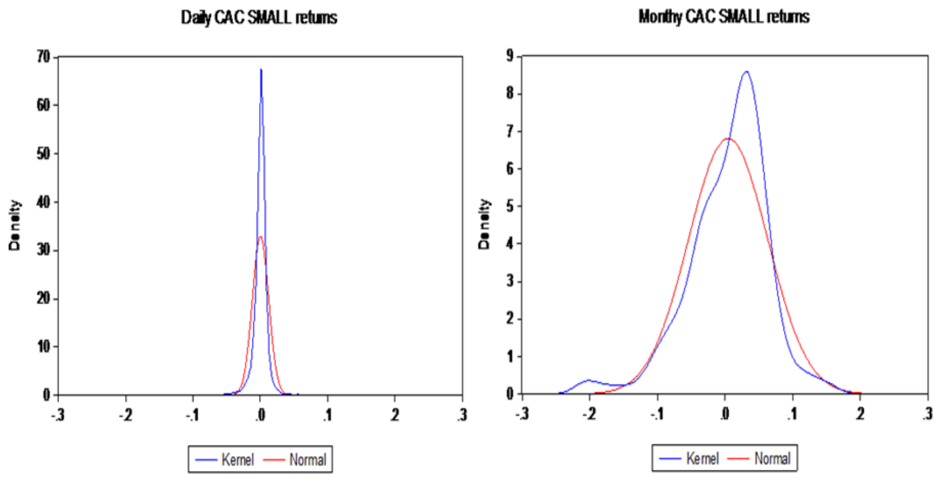
As seen in Table 3, the random walk hypothesis is clearly rejected. The BDS (Brock *et al.*, 1996) statistics are strictly greater than the critical value at 5% and the variance ratio is statistically different from one. In other words, the critical probabilities are less than 0.05 for sampling intervals of 2,4,8 and 16 days (see Table 4).

Figure 2. **Daily and Monthly CAC SMALL (logarithmic series and returns)**



Source: authors representation

Figure 3. **Kernel estimation of density**



Source: authors representation

Table 3. BDS test results on the series of returns

m	**Daily returns**		**Monthly returns**	
	BDS stat.	Prob.	BDS stat.	Prob.
2	15.315	0.000	3.207	0.001
3	20.066	0.000	3.507	0.000
4	22.654	0.000	4.069	0.000
5	25.133	0.000	3.960	0.000
6	27.714	0.000	4.103	0.000
7	30.980	0.000	4.178	0.000
8	34.755	0.000	4.331	0.000
9	38.950	0.000	4.626	0.000
10	43.787	0.000	5.130	0.000

Notes: The BDS statistics are calculated by the fraction of pairs method with ε equal to 0.7.
Source: authors calculations

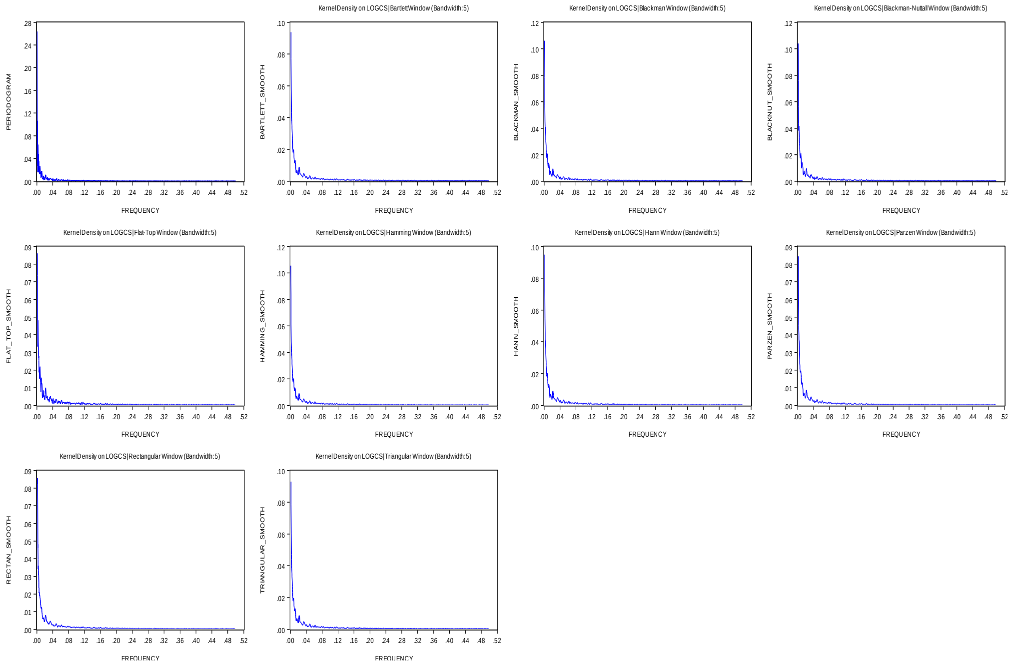
The BDS and the variance ratio tests generally bring out the presence of significant non-zero autocorrelations in the short term. These two tests lead us to reject the *i.i.d* hypothesis, but do not detect the presence of long-term dependence structure. Given this situation, we test the presence of autocorrelations by considering longer horizons. By plotting the periodogram of the series (see Figure 3 and 4) (with all smoothing windows), we note that the spectral density is concentrated around low frequencies and tends to infinity when the frequency tends to zero. This is a sign of long memory.

Table 4. Variance ratio estimates and test statistics of random walk hypothesis for the entire period (1999 ~ 2015)

Frequency	**Joint Tests**		**Value**	**df**	**Probability**
Daily returns	Max |z| (at period 2)*		5.093289	3231	0.0000
			Individual Tests		
	Period	Var. Ratio	Std. Error	z-Statistic	Probability
	2	0.510947	0.096019	-5.093289	0.0000
	4	0.268285	0.146368	-4.999149	0.0000
	8	0.139067	0.182828	-4.708983	0.0000
	16	0.064805	0.221936	-4.213814	0.0000
Monthly returns	Max |z| (at period 2)*		4.038245	202	0.0002
			Individual Tests		
	Period	Var. Ratio	Std. Error	z-Statistic	Probability
	2	0.649550	0.091592	-3.826195	0.0001
	4	0.349792	0.161012	-4.038245	0.0001
	8	0.192745	0.237391	-3.400524	0.0007
	16	0.099566	0.342735	-2.627197	0.0086

Notes: *Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom.

Figure 4. Periodograms of daily returns



Source: authors representation

From Table 5, it is clear that the daily and monthly series of CAC SMALL returns are generated by a long memory process. The values of Student statistic (with a power of 0.8) are strictly greater than the critical value at 5%. In addition, the memory parameter estimated by a Gaussian semiparametric method (Robinson and Henry, 1998) is positive and significant. The estimation result is very close to those found with the GPH (Geweke and Porter-Hudak, 1983) method. The presence of a long memory indicates that agents can anticipate their returns to a sufficiently long time horizon. Indeed, the observed movements appear as the result of lasting exogenous shocks, which affect the Paris stock market. The return will not come back to its fundamental value.

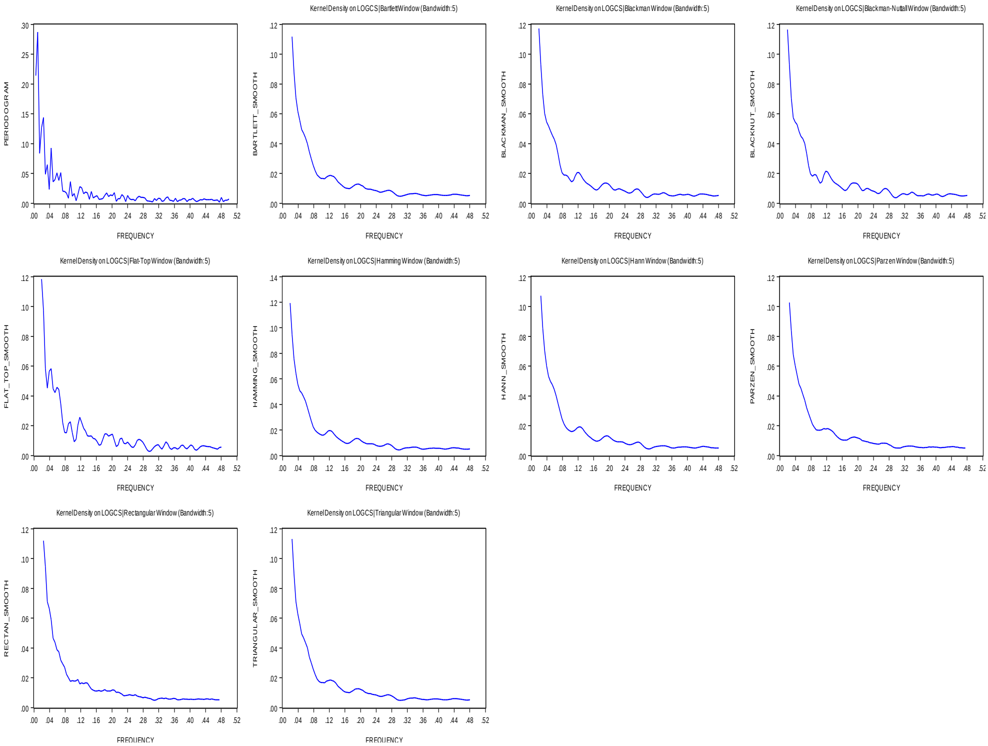
Table 5. Results from the ARFIMA (0,d,0) estimation using spectral methods on daily and monthly CAC SMALL returns

Frequency	**Daily returns**		**Monthly returns**	
Windows	Long memory parameter	Student statistics	Long memory parameter	Student statistics
GPH	0.1202	4.6094	0.0451	0.5150
Rectangular	0.1057	3.6741	0.1650	1.7063
Bartlett	0.1066	6.4209	0.1437	2.5751

Daniell	0.1066	5.2432	0.1478	2.1618
Tukey	0.1061	5.8546	0.1488	2.4414
Parzen	0.1068	7.1490	0.1317	2.6239
B-priest	0.1058	4.7508	0.1578	2.1072
Robinson-Henry method	0.1017	9.0831	0.2363	4.7545

Source: authors calculations

Figure 5. Periodograms of monthly returns



Source: authors representation

We propose, on the one hand, to test the possible presence of the day-of-the-week and the month-of-the-year effects on our daily and monthly financial series and, on the other hand, to verify whether this seasonality is real and not specious. The financial asset prices often exhibit heteroscedastic behavior with persistence. For this reason, we will study the conditional variance of daily and monthly CAC SMALL returns, to consider the possibility of asymmetric shocks and seasonality on volatility. In practical terms, we estimate four models: AR(1)-GARCH(1,1), AR(1)-SD-GJR-GARCH(1,1), SEMIFARMA(p,d,q)-SDGARCH(1,1)) and

SEMIFARMA(p,d,q)-SD-GJR-GARCH(1,1). For each model, we calculate both Akaike (1970) and Schwarz (1978) information criteria. After estimating the nonparametric deterministic trend, the optimal window and the cross-validation criteria by the kernel method based on the methodology of Nadaraya-Watson (1964) and Watson (1964)), the results of the estimation by the exact maximum likelihood method and corresponding t-statistics using the GED innovation distribution are displayed in Table 6.

Table 6. Maximum likelihood estimation - BFGS algorithm

Daily returns					Monthly returns				
Coefficients	AR(1)-GARCH(1,1)	AR(1)-SD-GJR-GARCH(	SEMIFARMA(0,d,2)-SDGARCH(1,1	SEMIFARMA(0,d,2)-SD-GJR-GARCH(1,1)	Coefficients	AR(1)-GARCH(1,1)	AR(1)-SD-GJR-GARCH(	SEMIFARMA(0,d,0)-SDGARCH(1,1	SEMIFARMA(0,d,0)-SD-GJR-GARCH(1,1)
Conditional mean equation									
γ	-	-	0.0015 (6.987)	0.001 (5.112)	γ	-	-	-	-
ϕ_1	0.2003 (4.209)	0.2104 (10.76)	-	-	ϕ_1	0.3475 (4.847)	0.3316 (4.275)	-	-
θ_1	-	-	0.060 (2.405)	-	θ_1	-	-	-	-
θ_2	-	-	0.051 (2.785)	2.013 (2.013)	θ_2	-	-	-	-
d_1	-	-	0.085 (4.269)	0.139 (10.960)	d_1	-	-	0.262 (4.091)	0.247 (2.287)
$\hat{h}_{opt}$	0.3418	0.3418	0.3418	0.3418	$\hat{h}_{opt}$	0.3524	0.3524	0.3524	0.3524
CV	0.6127	0.6127	0.6127	0.6127	CV	0.6254	0.6254	0.6254	0.6254
Conditional variance equation									
α_0	0.0120 (3.254)	0.0155 (3.185)	0.000001 (5.407)	0.000001 (6.322)	α_0	2.7443 (1.412)	3.6471 (2.067)	0.0002 (2.871)	0.0002 (2.107)
α_1	0.0997 (4.736)	0.0576 (3.046)	0.103 (10.446)	0.047 (5.552)	α_1	0.1044 (1.972)	0.0017 (0.0368)	0.131 (2.288)	0.017 (2.332)
β_1	0.8907 (40.86)	0.8852 (32.85	0.886 (126.873)	0.885 (136.873)	β_1	0.8110 (10.23)	0.7952 (10.82)	0.793 (10.810)	0.810 (11.660)
γ_1	-	0.0847 (2.898)	-	0.099 (6.041)	γ_1	-	0.1720 (2.946)	-	0.165 (2.045)
Monday	-	0.0211 (3.0012)	0.0458 (3.187)	0.0423 (3.412)	January	-	0.0059 (2.4656)	0.0048 (3.495)	0.0035 (3.698)

Tuesday	-	-0.0314 (-0.832)	-0.0871 (-1.045)	0.0894 (1.244)	February	-	0.0033 (0.1921)	-0.0002 (-0.126)	0.0004 (0.326)
Wednesday	-	-0.0018 (-0.5241)	-0.0133 (-0.841)	-0.0741 (-1.172)	March	-	-0.0204 (-1.4621)	-0.0006 (-0.378)	-0.0010 (-1.115)
Thursday	-	0.0722 (1.9912)	0.0724 (1.674)	0.0702 (1.357)	April	-	0.0008 (0.0432)	0.0023 (2.216)	0.0014 (2.145)
Friday	-	0.0359 (2.2147)	0.0423 (2.345)	0.0578 (2.675)	May	-	-0.0186 (-1.1069)	-0.000006 (-0.054)	0.0002 (0.418)
			-	-	June	-	0.0088 (0.4981)	0.0012 (1.087)	0.0005 (0.797)
			-	-	July	-	0.0082 (0.4645)	0.0016 (1.251)	0.0008 (1.325)
			-	-	August	-	0.0012 (0.1025)	0.0009 (0.889)	0.0007 (1.185)
			-	-	September	-	0.0223 (2.0419)	0.0057 (1.823)	0.0021 (2.329)
			-	-	October	-	-0.0048 (-0.3194)	-0.0028 (-1.074)	-0.0005 (-0.653)
			-	-	November	-	-0.0029 (-0.1603)	0.0016 (0.888)	0.0004 (0.541)
Jarque-Bera	44496.0	47327.0	53353.03	55774.17	Jarque-Bera	28.565	13.223	12.093	5.020
ARCH(1)	0.0291	0.0502	0.094	0.107	ARCH(1)	0.2437	0.5979	0.143	0.885
AIC / Schwarz	-6.8837 / -6.8743	-6.8911 / -6.8798	-6.920 / -6.905	-6.928 / -6.913	AIC / Schwarz	-2.9586 / -2.8770	-2.9591 / -2.8782	-2.951 / -2.870	-2.964 / -2.886
SSR	0.4659	0.4657	0.4656	0.4654	SSR	0.6443	0.6442	0.6445	0.6431
Log likelihood	11129.150	11142.020	11192.15	11204.67	Log likelihood	304.299	304.774	304.621	306.923
GED parameter	1.0736 (21.92)	1.0823 (21.73)	1.042 (43.646)	1.047 (45.806)	GED parameter	1.4840 (6.370)	1.5922 (6.556)	1.639 (6.750)	1.741 (5.736)

Notes: The Student statistics, CV: Optimal Cross Validation, $\widehat{h}_{opt}$: optimal bandwidth

Source: authors calculation

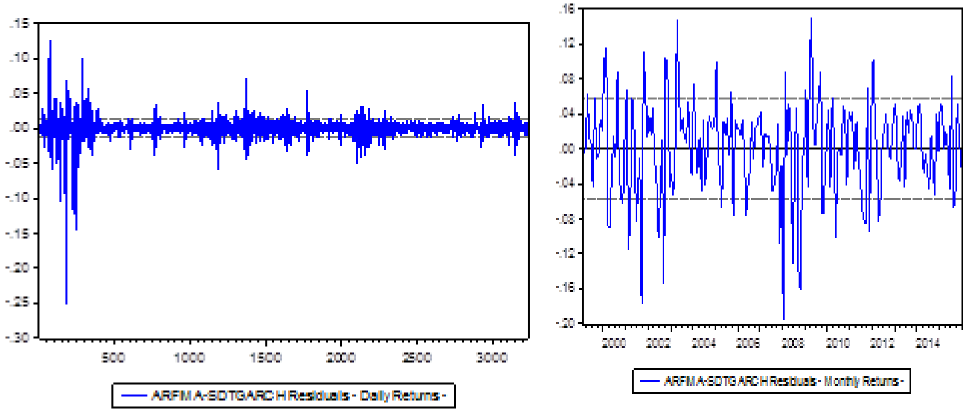
The results indicate that the daily and the monthly CAC SMALL series are characterized by a long memory: the estimated fractional integration parameter is significantly different from zero on the basis of the t-statistics. This result is consistent with the different spectral method estimators. Given that the daily and the monthly CAC SMALL returns are characterized by the presence of long-term dynamics in the equation of the mean and by heteroscedasticity, the SEMIFARMA-

SD-GJR-GARCH model is clearly superior to SEMIFARMA-SDGARCH, AR-GARCH and AR-SD-GJR-GARCH models because the information criteria are minimum for the SEMIFARMA-SD-GJR-GARCH model and the high significance of the α_1 and β_1 estimates under the SDGARCH and SD-GJR-GARCH models validate the presence of volatility clustering in the monthly and daily CAC SMALL returns. The results for the parameter γ confirm the presence of leverage effect at 5% in the SD-GJR-GARCH model. Furthermore, the leverage parameter is positive value hence confirming that bad news tend to increase volatility. The SDGARCH(1,1) performs worst due to its inability to capture the leverage effect feature of the CAC SMALL returns. Furthermore, the shape parameter estimate ν is highly significant and this leads to conclude that the distributions of the return series have fatter tails that the Gaussian distribution hence supporting the appropriateness of the GED distributions. In addition, it is observed that the coefficients of January, April and September are significant at the 5% level. For daily returns, some seasonal coefficients are also significant at 5%. In other words, the month of the year effect and the day of the week effect are present in the stock CAC SMALL returns. Moreover, the investors can trade on past information or they are in a position to predict the nature of stock prices based on the past information. By trading on this past information, they can earn abnormal returns.

The estimation results of the SEMIFARMA-SD-GJR-GARCH model seem that Friday has a greater effect than Monday at variance equation; this indicates that the weekend has greater volatility than the beginning of the week at the level of daily data. At the level of monthly data, January has a larger effect than the September and April effect. The sum of these two last coefficients is equivalent to the January effect coefficient (0.35=0.14+0.21). The dealers receive a large amount of information in the news and on the Internet during the weekend. In other words, companies may prefer to advertise bad news during the weekend. Moreover, the stock market may be affected by the abnormalities of other markets in light of the co-movement, the clients in both markets are similar in terms of their liquidity preferences and the information is asymmetric among dealers.

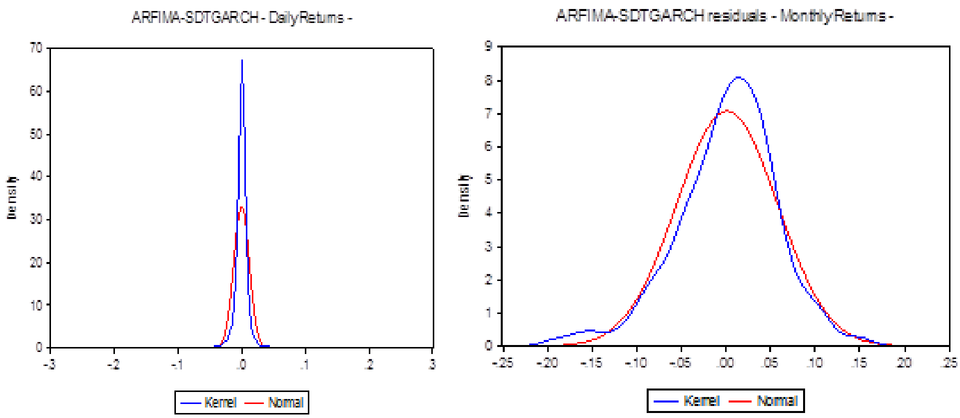
As shown in Table 6, the residuals of the models are characterized by the absence of conditional heteroskedasticity: the ARCH-LM statistics are strictly less than the critical value of χ^2 at 5%. It should be noted that the normality assumption of residuals is clearly rejected by the Jarque-Bera test for daily returns and accepted for monthly returns (see also Figure 6).

Figure 6. **Evolution of estimation residuals**



Source: authors representation

Figure 7. **Kernel estimation of density**



Source: authors representation

4. Forecasting performance

We perform the out-of-sample tests of forecasting accuracy using the minimum loss functions on AR-GARCH, AR-SD-GJR-GARCH, SEMIFARMA-GARCH, SEMIFARMA-SDGARCH and SEMIFARMA-SD-GJR-GARCH for GED distribution and the random walk model. The forecast evaluation measures used include mean square error (RMSE) and mean absolute error (MAE). The RMSE criterion is a square root of quadratic scoring rule which measures the average magnitude of the error and the MAE criterion is more robust to outliers since it does not make use of square.

Table 7. Comparison of predictive qualities (out-of--sample forecasts)

Frequency	**Equation**	**Horizon**	**Criteria**	**AR-GARCH**	**AR-SD-GJR-GARCH**	**SEMIFARMA-GARCH**	**SEMIFARMA-GJR-GARCH**	**SEMIFARMA-SD-GJR-GARCH**	**Random Walk**
Daily	Conditional mean	1	RMSE	0.0056	0.0057	0.00617	0.00619	0.00618	0.01270
			MAE	0.0056	0.0057	0.00617	0.00619	0.00618	0.01197
		2	RMSE	0.0072	0.0088	0.00639	**0.00632**	0.00634	0.01734
			MAE	0.0069	0.0080	0.00630	**0.00628**	0.00629	0.01721
		15	RMSE	0.0071	0.0070	0.00622	0.00620	**0.00617**	0.04681
			MAE	0.0063	0.0063	0.00519	0.00550	**0.00510**	0.04664
		30	RMSE	0.0066	0.0064	0.00573	0.00571	**0.00564**	0.06677
			MAE	0.0057	0.0053	0.00498	0.00484	**0.00473**	0.06591
	Conditional variance	1	RMSE	0.000038	0.000036	0.000042	0.000032	**0.000030**	-
			MAE	0.000038	0.000036	0.000042	0.000029	**0.000028**	-
		2	RMSE	0.000042	0.000057	0.000045	**0.000037**	0.000038	-
			MAE	0.000041	0.000047	0.000045	**0.000033**	0.000035	-
		15	RMSE	0.000041	0.000040	0.000048	0.000039	**0.000031**	-
			MAE	0.000039	0.000039	0.000042	0.000036	**0.000028**	-
		30	RMSE	0.000045	0.000044	0.000056	0.000043	**0.000038**	-
			MAE	0.000032	0.000033	0.000047	0.000038	**0.000031**	-
Monthly	Conditional mean	1	RMSE	0.0235	0.0231	**0.02212**	0.02216	0.02214	0.05912
			MAE	0.0235	0.0231	**0.02212**	0.02213	0.02213	0.05804
		2	RMSE	0.0185	0.0185	0.01757	0.01745	**0.01742**	0.08343
			MAE	0.0184	0.0185	0.01751	0.01741	**0.01733**	0.08250
		15	RMSE	0.0422	0.0422	0.01656	**0.01612**	0.01628	0.20093
			MAE	0.0372	0.0372	0.01549	**0.01539**	0.01627	0.19770

	30	RMSE	0.0408	0.0408	0.01402	0.01400	**0.01399**	0.25342
		MAE	0.0350	0.0350	0.01389	0.01371	**0.01365**	0.22997
Conditional variance	1	RMSE	0.0025	**0.0022**	0.00241	0.00239	0.00240	-
		MAE	0.0025	**0.0022**	0.00241	0.00239	0.00240	-
	2	RMSE	0.0036	0.0032	0.00254	0.00241	**0.00238**	-
		MAE	0.0036	0.0032	0.00254	0.00241	**0.00238**	-
	15	RMSE	0.0018	0.0028	0.00236	0.00226	**0.00225**	-
		MAE	0.0015	0.0026	0.00217	0.00212	**0.00108**	-
	30	RMSE	0.0033	0.0032	0.00268	0.00257	**0.00254**	-
		MAE	0.0030	0.0039	0.00259	0.00231	**0.00212**	-

Source: authors calculations

Table 7 contains statistical comparisons of out-of-sample forecasts provided by the different models. It is observed that the RMSE and MAE criteria generally give the same results. We find that the five models outperform the random walk model in all forecasting time horizons.

Table 8. Comparing predictive accuracy: Diebold-Mariano test

Test of equal accuracy	Frequency	S_1	S_2	S_3	MGN
SEMIFARMA-SD-GJR-GARCH versus SEMIFARMA-GJR-GARCH	Daily	- 1.60 (0.11)	- 9.25 (0.00)	- 3.75 (0.00)	- 15.87 (0.00)
	Monthly	- 4.43 (0.66)	-8.57 (0.00)	- 6.82 (0.00)	-10.54 (0.00)
SEMIFARMA-SD-GJR-GARCH versus Random walk	Daily	- 0.61 (0.11)	-5.45 (0.00)	-3.23 (0.00)	-9.01 (0.00)
	Monthly	- 0.30 (0.77)	- 3.98 (0.00)	- 2.72 (0.01)	- 10.23 (0.00)

Note: The p-values are given in parentheses. S_1: Asymptotic test statistic, S_2: Sign test statistic, S_3: Wilkoxon test statistic, MGN: Morgan-granger-Newbold test statistic,. A positive (negative) sign of the statistics implies that model B dominates (is dominated by) model A.The prediction horizon used is 30. These tests are based on absolute forecast errors.
Source: authors calculations

The SEMIFARMA-SD-GJR-GARCH model tends to have better predictive results compared to AR-GARCH, AR-SD-GJR-GARCH, SEMIFARMA-GJR-GARCH and SEMIFARMA-GARCH in 15, 30 days for daily and monthly CAC SMALL return series. Moreover, the values of RMSE and MAE criteria decrease with the prediction horizons because all the last three models take into account the long-term memory in the conditional mean equation and therefore completely neglect the long-term memory in the conditional volatility, considering that the criteria increase with the prediction horizons. In other words, the predictive power for daily and monthly CAC SMALL returns reflects the possibility to forecast up to the longest horizon.

In order to test the statistical significance of the forecasting improvements of SEMIFARMA-SD-GJR-GARCH predictions over the SEMIFARMA-GJR-GARCH, on one hand, and the random-walk on the other hand, we can use also a battery of tests based on loss functions: the asymptotic test, the sign tests, Wilcoxon's test and the Morgan-Granger-Newbold test (Diebold and Mariano 1995). The null hypothesis is the equal predictive accuracy of the two models. The results are reported in Table 9.

As seen in Table 8, the p-values clearly indicate that the null hypothesis of equal accuracy of the three models is strongly rejected. It is observed that different predictive accuracy is accepted because the p-values are less than 0.05, which means that, in this case, the SEMIFARMA-SD-GJR-GARCH model beats the SEMIFARMA-GJR-GARCH and the random walk process. The Diebold-Mariano statistics are, in most cases, significant, meaning that there is a difference in the forecasts computed from the GJR-GARCH and SD-GJR-GARCH models. A negative sign of the statistics implies that the GJR-GARCH model without seasonal dummies is dominated by GJR-GARCH model with seasonal dummies (SD-GJR-GARCH). The results indicate that the day-of-the-week and the month-of-the-year effects detected on volatility seem to improve the volatility forecasts. Indeed, the sign of the statistics is negative, implying that the day-of-the week and the month-of-the-year effects observed on volatility provide a better volatility forecast.

We further probe these results by using the estimation results to compute out-of-sample value-at-Risk for the long and short trading position for confidence levels 95%, 97.5%, 99%, 99.5% and 99.75%. The results presented in Table 9 report the success/failure ratio and the Kupiec likelihood ratio (Kupiec, 1995). The LR statistics has the distribution χ^2 with one degree of freedom. For the SEMIFARMA model with SD-GJR-GARCH errors, the null hypothesis of the test is not rejected, either in case of underestimating potential loss or in case of overestimating VaR for the short and long positions, which means that the null hypothesis of correct unconditional coverage can be accepted for all the confidence levels. However, the Kupiec test results indicate that the out-of-sample VaR forecast for daily CAC SMALL returns obtained by the AR-GJR-GARCH model gives unsatisfactory results and consequently fails this test at 95%confidence level for short and long trade positions. It seems that the SEMIFARMA-SD-GJR-GARCH is appropriate for capturing volatility clustering and seasonality for both negative (long Value-at-Risk) and positive returns (short Value-at-Risk) for daily and monthly series.

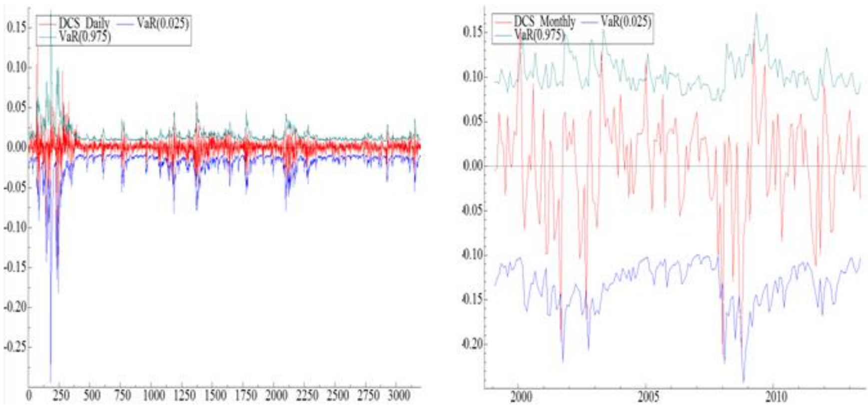
Table 9. Out-of-Sample Value-at-Risk and backtesting with GED distribution

Frequency	Model	Position	Quantile	Success	Kupiec	Prob.
Daily	AR-GJR -GARCH	Short positions	0.95	0.9406	8.7033	0.0031
			0.975	0.9724	1.3099	0.2524
			0.99	0.9877	2.4292	0.1190
			0.995	0.9951	0.0288	0.8650
			0.9975	0.9981	1.0449	0.3066
		Long positions	Quantile	Failure	Kupiec	Prob.
			0.05	0.0485	0.2342	0.6283
			0.025	0.0229	0.8830	0.3473
			0.01	0.0088	0.6822	0.4088
			0.005	0.0032	3.6201	0.0570
			0.0025	0.0012	4.1177	0.0424
	SEMIFARMA-SD-GJR-GARCH	Short position	Quantile	Success	Kupiec	Prob.
			0.95	0.9446	2.9021	0.0884
			0.975	0.9746	0.0266	0.8703
			0.99	0.9893	0.2193	0.6395
			0.995	0.9963	2.0945	0.1478
			0.9975	0.9983	1.8061	0.1789
		Long positions	Quantile	Failure	Kupiec	Prob.
			0.05	0.0541	1.7531	0.1854
			0.025	0.0271	0.9371	0.3330
			0.01	0.0088	0.6822	0.4088
			0.005	0.0036	2.0945	0.1478
			0.0025	0.0016	1.8061	0.1789
Monthly	AR-GJR -GARCH	Short positions	Quantile	Success	Kupiec	Prob.
			0.95	0.9479	0.0147	0.9034
			0.975	0.9653	0.5948	0.4405
			0.99	0.9826	0.7723	0.3794
			0.995	0.9942	0.0201	0.8871
			0.9975	0.9942	0.5432	0.4611
		Long positions	Quantile	Failure	Kupiec	Prob.
			0.05	0.0462	0.0526	0.8184
			0.025	0.0173	0.4655	0.4950
			0.01	0.0057	0.3668	0.5447
			0.005	0.0000	-	0.0000
			0.0025	0.0000	-	0.0000
	SEMIFARMA-SD-GJR-GARCH	Short positions	Quantile	Success	Kupiec	Prob.
			0.95	0.9422	0.2116	0.6454
			0.975	0.9653	0.5948	0.4405
			0.99	0.9768	2.1956	0.1384
			0.995	0.9942	0.0201	0.8871
			0.997	0.9942	0.5432	0.4611
		Long positions	Quantile	Failure	Kupiec	Prob.
			0.05	0.0462	0.0526	0.8184
			0.025	0.0173	0.4655	0.4950
			0.01	0.0057	0.3668	0.5447
			0.005	0.0034	2.0944	0.1479
			0.0025	0.0012	1.8063	0.1788

Source: authors calculations

Figure 7 illustrates the relation of the Value-at-risk with the return of stock prices. The upper line is the maximal amount that can be lost with a confidence level 97.5% over the period of time taken into consideration, when business events are not favourable for the business activity. The calculation of VaR by use of the SEMIFARMA-SD-GJR-GARCH model also has the advantage of forecasting the values of the VaR in the future. If the other factors remain constant, then the SD-GJR-GARCH model gives a very high level of approximation with the real values of the VaR.

Figure 8. **Out-of-Sample Value-at-Risk forecasts**



Source: authors representation

Given that the daily and monthly CAC SMALL returns are characterized by the presence of long memory dynamics in the equation of the mean and by the seasonal and asymmetric effects in the conditional volatility, the SEMIFARMA-SD-GJR-GARCH modelling allows computation of better forecasts than the other models and the random walk. The returns are long-term predictable. The agents can anticipate their returns on a long time horizon. Indeed, the observed movements appear as the result of lasting exogenous shocks, which affect the Paris stock market. The CAC SMALL returns will not come back to their previous fundamental value and the shock will be persistent in the long term. This suggests that, due to the long-term predictability of returns, it will be possible to establish remunerative strategies on the Paris stock market a priori. In addition, the two series are characterized by the existence of Day-of-the Week (DOW) and January effects in the volatility. There is an asymmetric impact of positive and negative information at the level of future variance. The weak efficiency assumption of financial markets seems violated for daily and monthly CAC SMALL returns.

The main result of this paper is that there is a long memory in the series of small capitalization returns and predictability of returns so it can be said that the market of small capitalization in Paris is inefficient and that, on the other hand, the presence of the weekend and January effects is evidence of bias in returns. The interpretation of these biases is due to the irrational behaviour of investors and is the subject of behavioural finance. Finally, the small capitalization market in Paris is inefficient at the weak level and has biases.

Conclusions

Behavioural finance is currently one of the hot topics in the financial literature; behavioural finance criticizes the efficiency of financial market hypothesis and itis concerned with the behaviour of the investor, which results in abnormalities in prices and returns. This paper has investigated the presence of seasonal and asymmetric effects on the volatility of CAC small capitalization. The empirical results provide evidence of nonparametric deterministic trends, long-range dependence and short dependence in daily and monthly CAC SMALL index as well as seasonal asymmetric time-varying GARCH errors. These classes of models have been estimated by the exact maximum likelihood method, taking into account the phenomenon of seasonality and asymmetry persistence for the conditional variance. Our results indicate that informational shocks have lasting effects on returns and transitory effects on volatility. Furthermore, the seasonal effect has an asymmetric impact on the conditional volatility. The day-of-the-week and the month-of-the-year effects detected on volatility have given better volatility forecasts, implying that it is necessary to integrate these effects in trading strategies. Indeed, the existence of seasonal and asymmetric effects of positive and negative shocks could be interesting only if their incorporation in a model improves the forecasting accuracy of volatility. In this case, investors could develop a trading strategy to benefit from these seasonal regularities. Consequently, the existence of anomalies in small-cap equity returns in Paris stock markets reveals that the stock market is inefficient and the rationality is limited, as there are arbitrage opportunities. This result requires an in-depth study of the behaviour and sentiment of investors.

For the future research of this issue, we can propose to re-study the problematic of this paper with the comparison between small and large companies capitalization and highlight the size premium. Apply the model of this study to small-cap in different countries. Study the relationship between the small-cap index and the investor sentiment index can be tested.

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